Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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в	г	е	ё	ж	и	й	у	х	ц	ч	ш	щ	ъ	ы	ь	э	ю	я
v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ɨ	′	è	yu	ya

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March 1950 Vol. 3, No. 3

Communications

New Periodical

Zeitschrift für angewandte Mathematik und Physik (ZAMP, Journal of Applied Mathematics and Physics) began January 1950, will appear six times a year and publish about 80 pages of surveys, original papers, brief reports and book reviews in German, French, Italian, or English. The editorial board comprises Ackeret, Baumann, Niggli, Scherrer, Stiefel, Stüssi, Ziegler; the editor is R. Sänger: all associated with the Swiss Federal Institute of Technology (ETH) at Zürich. The publisher is Verlag Birkhäuser at Basel, Switzerland.

Concerning Rev. 1471, November 1949, "Cylindrical journal bearings under nonstationary loading," by Hanns Herbert Ott (Zürich, 1948).

In the review of this paper, M. C. Shaw comes to the conclusion that the derived differential equation for the pressure in the oil film between two moving surfaces is not correct and gives in its place a different equation.

In the paper discussed, the equation in question is derived by application of the equation of continuity in its integrated form, while M. C. Shaw develops his equation by means of the equation of continuity in its differential form. He obtains a different result because the boundary conditions are not correctly applied. By exact consideration of the boundary conditions one also arrives from the differential continuity equation at the relation which has been obtained in the paper discussed. In its general form, when the velocities V_1 and V_2 are considered as dependent on x, the correct equation reads

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) - 6\eta\,(V_1 + V_2)\frac{\partial h}{\partial x} - 6\eta h\frac{\partial}{\partial x}(V_1 + V_2) - 12\eta\frac{\partial h}{\partial t} = 0.$$

First, the equation of Shaw differs from the aforementioned in that his signs of V_1 and V_2 are interchanged in the second term. It is evident that this cannot be correct because both surfaces are equivalent and their velocities therefore must have the same sign. Also the dimensions of the last term in the equation of Shaw are not consistent. It can be shown that the factor η is lacking.

In his review M. C. Shaw states further that the dependence of V_1 and V_2 on x cannot be discarded as was done in the discussed work. It can easily be shown that in the case of the cylindrical journal bearing for which the equation is derived, $h\partial(V_1 + V_2)/\partial x$ is approximately to $\partial h/\partial t$ as the radial clearance Δr is to the radius r of the journal. The term $6\eta h\partial(V_1 + V_2)/\partial x$ therefore can be discarded without any considerable error.

Hanns Herbert Ott, Switzerland

The basic difficulty with equation (8) appears to be due to the author's use of the Eulerian point of view in part of the derivation, and the Lagrangian point of view in the remainder of the derivation. Equation (2) is the equation of equilibrium written relative to a moving particle of fluid, i.e., according to the Eulerian method usually used in fluid mechanics. However, the continuity equation in the form of equation (6) is written relative to a fixed volume in space, i.e., according to the unusual Lagrangian method. Thus, in applying equation (8), certain terms must be interpreted from the unconventional Lagrangian point of view. If, however, the differential form of the continuity equation is written relative to an infinitesimal volume moving with the fluid, only the Eulerian method is involved and the following equation results:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) - 6\eta(V_1 - V_2)\frac{\partial h}{\partial x} - 6\eta h\frac{\partial}{\partial x}(V_1 + V_2) - 12\eta W = 0$$

(the last term of this equation was incorrectly printed in Rev. 2, **1471**) where W is the relative velocity of approach of the two surfaces in the y direction. When this equation is applied to a journal bearing, $W = V_2\partial h/\partial x + \partial h/\partial t$ and the author's equation (8) are obtained except for the term $6\eta h\,\partial(V_1 + V_2)/\partial x$. As the author points out in his comment, the missing term turns out to be negligible. It should therefore be made clear that the foregoing discussion should in no way alter the results obtained by the author or any of his discussion beyond paragraph 2, page 11 of the paper.

Milton C. Shaw, USA

Theoretical and Experimental Methods

(See also Revs. 420, 421, 435, 466, 524, 583)

394. R. S. Silver, Philosophy of applied science, Research Lond. 2, 149–153 (Apr. 1949).

Pure and applied science are compared and discussed from the viewpoints of education, training, experience, objectives and application. A distinction is drawn between the emphasis on abstract generality in pure science, culminating in mathematics, and the concrete, experience-conditioned, semi-intuitive aspects of applied science as exemplified by the skilled craftsman or engineer. The one analyzes; the other synthesizes. The author concludes with a plea for a broader recognition of these facts, and for development of a statistics capable of handling more complex concepts than number. He advocates a deliberate awareness of the procedure and significance of acquiring practical experience in the effective application of theoretical knowledge or, in brief, wider training in the why and how of the methods which made "Operational Research" so effective during the war.

K. W. Miller, USA

395. P. R. Garabedian and M. Schiffer, Identities in the theory of conformal mapping, Trans. Amer. math. Soc. 65, 187–238 (Mar. 1949).

The authors undertake a systematic study of the relations between the various types of canonical conformal maps of a given multiply-connected two-dimensional region, which are of importance both in the mathematical theory of conformal mapping and in applications to the theory of incompressible fluids. By consistent application of a contour integration method which utilizes the geometric properties of the canonical domains, a large

number of identities—many of them new—are derived. An essential feature is the prominent role played by various kernel functions which are shown to be closely interrelated with the various domain functions of theoretical and practical interest. Since kernel functions can be computed by means of a comparatively simple algorithm, these relations may be used for the effective determination of canonical mapping functions and, thus, for the construction of incompressible flow patterns.

Zeev Nehari, USA

396. J. Meixner, Lamé's wave functions of the ellipsoid of revolution, Nat. adv. Comm. Aero. tech. Memo. no. 1224, 102 pp. (Apr. 1949) [transl. from Zent. wiss. Berichtswesen Berlin, Forschungsber. no. 1794 (1943)].

The three-dimensional wave equation is treated by separation of variables in rotationally symmetric elliptic coordinates. The solutions of an ordinary differential equation so arising are then investigated. These solutions are developed in terms of series of spherical functions and also in terms of cylinder functions. Certain identities among these solutions are given; the convergence of the series representations is considered; various developments (or calculation methods) are given for finding the coefficients; and the eigenvalue problems are discussed. Several tables listing parameters of interest are included. George Carrier, USA

397. J. L. Synge and A. Schild, Tensor calculus (Mathematical Expositions no. 5), University of Toronto Press, Toronto, 1949, 324 pp. Cloth, 5.7 × 8.5 in., $6.

This book is intended for the mathematician, but the mechanologist who wants to learn tensor calculus quickly and painlessly can use the following chapters of it:

I. Spaces and tensors (24 pp.). II. Basic operations in Riemannian space (58 pp.). V. Applications to classical dynamics (51 pp.). VI. Application to hydrodynamics, elasticity, and electromagnetic radiation, first two sections (27 pp.). Ed.

398. Edmondo Morgantini, Theory of nomograms with aligned points and rectilinear correspondences between forms of first species (in Italian), Rend. Sem. Mat. Univ. Padova 16, 3–72 (1947).

The object of the memoir is to give an exposition of the theory from an original and geometrical point of view. Chapter I sketches in projective terms the theory of the nomogram of genus zero. Chapter II treats the general alignment nomogram, and extends it by replacing the straight transversals by a double infinity of curves. Chapter III treats the situation where two or more unknowns are determined as functions of two independent variables by the points in which a straight transversal cuts four or more straight supports. Chapter IV treats the case where one unknown is determined as a function of three variables by two straight transversals which cut four straight supports and meet on a straight hinge, and chapter V develops a theory of quadrilinear correspondence which gives a necessary and sufficient condition that an equation be representable by such a nomogram.

Courtesy of Mathematical Reviews J. M. Thomas, USA

399. Arthur Sard, Best approximate integration formulas; best approximation formulas, Amer. J. Math. 71, 80–89 (Jan. 1949).

In numerical evaluation the integral is usually approximated by a linear combination of the values of equally spaced ordinates. The ordinate values are weighted according to some particular scheme, as the trapezoidal rule, Newton-Cotes formula, etc. The weighted sum of the ordinates yields an approximation which differs from the correct value by a remainder which may be expressed by $R(x) = {}_a\int^b x^{(n+1)}(t)k(t)dt$, where the $(n+1)$th derivative of the function $x(t)$ is multiplied by the weighting function $k(t)$. The author defines the best approximation as that for which the integral $J = {}_a\int^b k^2(t)dt$ is a minimum. This appraisal differs from that obtained by application of the theorem of the mean in that the latter minimizes the absolute value of the weighting function $k(t)$.

The author has calculated and tabulated coefficients for approximation formulas based on the minimum rms error criterion for up to seven ordinates and to third-degree approximation polynomials. In the derivation and proof, relations are developed which extend the range of application of the tables.

Vincent Salmon, USA

400. N. Zeilon, On the numerical calculation of eigenvalues (in Italian), R. C. Accad. Lincei 6, 52–60 (Jan. 1949).

A new method for computing the eigenvalues of a positive definite symmetric kernel is presented. In the development of the method, another well-known iterative procedure, employing the minimum property of the eigenvalues, is first reviewed. The latter procedure yields a monotonically decreasing sequence of positive numbers, $\lambda^{(\alpha)}$, whose limit is the first eigenvalue λ_1. It is shown that the rapidity of convergence of the sequence $\lambda^{(\alpha)}$ depends essentially upon the smallness of the ratio of the first two eigenvalues, λ_1/λ_2.

The sequence $\lambda^{(\alpha)}$ is then employed to derive two other sequences, both of which are more rapidly convergent than the original to the same limit λ_1. One of these sequences,

$$\Lambda^{(\alpha)} = \frac{[\lambda^{(\alpha+1)} - \lambda^{(\alpha+2)}][\lambda^{\alpha}\lambda^{(\alpha+2)} - \{\lambda^{(\alpha+1)}\}^2]}{\lambda^{(\alpha)} - 2\lambda^{(\alpha+1)} + \lambda^{(\alpha+2)}}$$

is also monotonically decreasing. The other,

$$\mu^{(\alpha)} = \frac{1}{1-s}[\Lambda^{(\alpha)} - s\lambda^{(\alpha+1)}]$$

becomes monotonically increasing after some member of the sequence, depending upon the choice of an arbitrary small number s. The two sequences thus define the limits of error in computing an eigenvalue.

The accuracy of the method is illustrated by its application to two boundary-value problems, the harmonic oscillator, and the vibrations of an elliptic membrane. Louis Landweber, USA

401. P. Matthieu, On the iteration method of Picard-Lindelöf for the approximate solution of ordinary differential equations (in German), Elemente Math. 4, 34–42 (Mar. 15, 1949).

This survey paper describes the analytical and graphical application of Picard's iteration method to the approximate solution of differential equations. Estimates of the error are given; extensions of the method to higher-order equations as well as systems of equations are mentioned; and there is a discussion of the extrapolation methods of Adams as a means of improving the convergence. Vincent Salmon, USA

402. Edmund Callis Berkeley, Giant brains or machines that think, John Wiley & Sons Inc., New York; Chapman & Hall, Ltd., London, 1949, xvi + 270 pp. Cloth, 8.3 × 5.6 in., 95 figs., $4.

This is a very popular and entertaining presentation of the elements of automatic computing machinery. The automatization of basic digital operations, and the operation of punch-card machines are discussed first. Then the MIT Bush-Caldwell analyzer, Harvard's IBM automatic sequence-controlled calculator, Moore School's Eniac, Bell Laboratories' general-purpose relay calculator are described in a truly intelligible way.

After a discussion of the Kalin-Burkhart logical-truth calculator, the newer devices (magnetic wire and tape, mercury tank, electric storage tube) are mentioned. The last two chapters deal with some bold vistas. A probably complete analytical bibliography (27 pp.) will be of value even to experts. Ed.

403. L. F. Hausman and M. Schwarzschild, Automatic integration of linear sixth-order differential equations by means of punched card machines, Rev. Sci. Instruments 18, 877–883 (1947).

The use of punched-card-relay calculators for the solution of systems of ordinary linear differential equations of total order six or less is described in considerable detail. Appropriate formulas are given for predicting and correcting the values for the $(n + 1)$th step on the basis of the values up to the nth step. These formulas are automatically calculated by the machine. The method of putting the values of the coefficients and the starting values on the proper cards is explained. The problem of very large or very small values is discussed and appropriate transformations are suggested. Finally, the results in an actual numerical example are given. W. E. Milne, USA

404. Göran Borg, On a Liapounoff criterion of stability, Amer. J. Math. 71, 67–70 (Jan. 1949).

This paper refines a result of Liapounoff concerning the differential equation

$$y'' + \phi(x)y = 0, \qquad \text{(L)}$$

where $\phi(x)$ is of period π. The following theorem is proved:

If in (L) $\phi(x)$ is continuous and $\phi(x + \pi) = \phi(x)$, and if

$$\int_0^\pi \phi(x)dx \geqslant 0 \text{ and } \phi(x) \not\equiv 0, \qquad (1)$$

$$\int_0^\pi |\phi(x)|dx \leqslant 4/\pi, \qquad (2)$$

then (L) is stable, which in this case means that all its solutions are bounded.

It is further shown that these conditions are the "best possible" in the sense that there are unstable differential equations (L) for which either (1) or (2) is violated. Condition (1) here replaces the Liapounoff condition that $\phi(x) \geqslant 0$ and $\phi(x) \not\equiv 0$, thus giving a much less stringent sufficient condition for stability.

The reviewer observes that (1) and (2) can be easily violated by both stable and unstable equations whose characteristic exponents are not logarithms of real numbers, which is really the case treated here. Felix Rosenthal, USA

405. Aloys Herrmann and Jean-Marie Souriau, A stability criterion deduced from Sturm's theorem (in French), C. R. Acad. Sci. Paris 228, 1183–1184 (Apr. 4, 1949).

A criterion is established to determine how many roots of the polynomial $P(x)$ have, respectively, positive, vanishing, and negative imaginary parts. The chain of Sturm polynomials $P_1(x), P_2(x), \ldots, P_k(x)$ is set up, the first two being, respectively, the real and imaginary parts of $P(x)$. From this chain the desired criterion is derived, with special reference to the question of stability. E. F. Masur, USA

406. W. G. Brombacher, Some problems in the precise measurement of pressure, Instruments 22, 355–358 (Apr. 1949).

Indicating instruments can be classified either by the type of pressure-sensitive element used, or by the range of pressures to be measured. Since many designs are limited to a definite pressure range, the author examines various instruments as to practicality and accuracy for certain ranges of pressures to be measured. An arbitrary classification lists (1) absolute pressure ranges: (a) vacuum, below 1 mm mercury, (b) 0.5 to 800 mm of mercury, (c) 1 to 3 atm, (d) above 3 atm; (2) differential pressures: (a) .001 to 1 mm of mercury, (b) up to 100 in. of mercury, (c) high pressures 50 to 50,000 psi, (d) high pressures 50,000 to 125,000 psi, (e) high pressures, above 125,000 psi.

Some of the instruments discussed are the mercurial barometer, the aneroid barometer, nesting capsules, the Matheson-Eden manometer, the Puddington manometer, the piston gage or deadweight tester, the multiple mercurial barometers.

This article is of value to those who are concerned with the accurate measurement of pressures, and wish to know of some of the equipment available, and their limitations. Frederick K. Teichmann, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 397, 404, 405, 416, 421, 423, 425, 430, 566)

407. A. Reuschel, On a kinematic construction principle for the determination of the curvature of curves and envelopes (in German), Öst. Ingen.-Arch. 3, no. 1, 9–23 (Jan. 1949).

The paper shows that it is possible to determine the center of curvature of any plane curve by constructing a constrained plane mechanism, one point of which traverses the curve which is located on a second member. The center of curvature of any point on the curve is located at the relative instantaneous center of the link containing the curve, and the link containing the point which traces the curve. A series of seven theorems are developed concerning the relative instantaneous centers. These theorems are based on the fact that the locus of the center of curvature of the curve (its evolute) is also the locus of the instantaneous center of the two members of the mechanism containing the curve and the tracing point. A general procedure is developed and its application to conic sections, exponential curves and sine curves is illustrated by several examples. The mechanism, of course, need not actually be constructed to utilize the graphical principles involved. William B. Stiles, USA

408. I. I. Artobolevskiĭ, Mechanisms for enveloping ellipses (in Russian), Doklady Akad. Nauk SSSR 65, no. 4, 453–456 (Apr. 1949).

The paper considers the following linkage: A and C are fixed pivots, AB and AD are two rigid bars, B and D are two crossheads, BCD is a right angle. It is shown that the perpendiculars BE (to BC) and DE (to DC) are always tangent to the same ellipse.

Three more similar mechanisms are described. A fifth one contains two straight lines enveloping two confocal ellipses. Parabolas and hyperbolas can be similarly enveloped. A. W. Wundheiler, USA

409. F. M. Dimentberg, The finite displacement of a spatial four-linkage with cylindrical pairs and the case of passive constraints (in Russian), Prikl. Mat. Mekh. 11, 593–602 (1947).

A four-bar spatial linkage in which the links are free to slide along three of the hinges is movable with one or more degrees of freedom. The constant parameters of this system are the lengths of the links (a length is the perpendicular distance between the hinges in a link) and the twists (a twist is the angle between the hinges in a link). The variables are the angles between the links. The relation between the variables is derived in terms of the parameters by means of vectors and finally expressed in analytic form.

Under special conditions, as in the case of a Bennett linkage in which the opposite lengths are equal and the opposite twists are

equal, but the hinges do not intersect, motion of the linkage is possible only when the lengths are proportional to the sines of the twists. In this case the bars of the linkage do not need to make use of the freedom to slide along the hinges. Such a condition is called a passive constraint. Other special cases in which passive constraints exist are the spherical linkage in which all the hinges intersect in a point, the plane linkage, and various degenerate cases such as the three-bar and the double two-bar.

M. Goldberg, USA

410. Jean Chazy, A course in rational mechanics (Cours de mécanique rationnelle), vols. I and II, Paris, Gauthier-Villars, 1947, vol. I: 482 pp., 189 figs.; vol. II: 511 pp., 173 figs. Paper, 6.5 × 10 in., \$5.80.

This is the third edition of a well-known treatise on theoretical mechanics which first appeared in 1933. Volume I deals with the dynamics of a single mass point, whereas volume II is devoted to the dynamics of systems of particles. The new edition, with the exception of minor revisions and additions, closely adheres to the original version. At the end of the first volume a number of new exercises have been included.

Ed.

411. I. S. Arzhaníkh, The vortex principle of analytical dynamics (in Russian), Doklady Akad. Nauk SSSR 65, no. 5, 613–616 (Apr. 1949).

This is a summary of the author's earlier paper [Akad. Nauk Uzbek SSR, 1948]. In familiar notations, the much-derived equation,

$$\delta(\Sigma p_i q_i - T) = \Sigma Q_i \delta q_i - \Sigma(\delta p_i \dot{\delta} q_i - dp_i \delta q_i),$$

is a form of d'Alembert's principle. The author calls

$$\partial p_i/\partial q_k - \partial p_k/\partial q_i$$

the "vortex of impulses," and the above equation "the vortex principle of dynamics."

A. W. Wundheiler, USA

412. Edward Kasner and John De Cicco, Generalization of Appell's transformation, J. Math. Phys. 27, 262–269 (Jan. 1949).

In the first part of this paper the authors consider the trajectories of a particle moving in a plane in what they call a generalized field of force, i.e., a field in which the force depends upon the position and direction of motion of the particle. They determine the contact transformations in the plane which, when associated with suitable transformations of the time of the form

$$dT = dt/F(x, y, p),$$

send all families of ∞^3 trajectories into other such families. (In this case the contact transformations turn out to be extended collineations.) They also determine the transformations which send a single family of trajectories into another such family. In the latter parts of the paper they give some similar results relating to various other families of curves, including families of curvature trajectories and families of generalized brachistochrones.

Courtesy of Mathematical Reviews L. A. MacColl, USA

413. V. V. Nemítskii, On the theory of orbits of general dynamic systems (in Russian), Mat. Sborn. 23, no. 2, 161–186 (Sept. 1948).

This paper contains proofs or results announced by the author in C. R. Acad. Sci. URSS 53, 491–494 (1946).

Ed.

414. Aurel Wintner, On linear repulsive forces, Amer. J. Math. 71, 362–366 (Apr. 1949).

The author considers a linear, nonconservative, reversible system of n degrees of freedom. The kinetic energy is of the classical type, so that by normalization the n inertial coefficients can be taken as +1. The Lagrangian equations are then assumed to be $x'' - F(t)\,x = 0$, where $F(t)$ is a real, symmetric, nonnegative definite, n-rowed matrix which is a continuous function of t, and x is the column matrix of the coordinates. The system thus generalizes to n dimensions the case of a mass-point with one degree of freedom, subject to a repulsive force function of time. The author proves that the system has n linearly independent solutions bounded as $t \to \infty$; more precisely, if $r(t) = x(t)^2$, he proves that $\lim r(t)$ exists and $\lim r'(t) = 0$, as $t \to \infty$. This result cannot be improved, for there exist n linearly independent solutions satisfying $r(t) \to \infty$ as $t \to \infty$ (compare the trivial conservative case $x'' - k^2 x = 0$).

P. LeCorbeiller, USA

415. E. A. Chudakov, Influence of lateral elasticity of wheels on the stability of an automobile against skidding (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk no. 10, 1635–1645 (Oct. 1948).

[See Rev. 1, **579**.] The author starts by giving a graph η_2 vs. γ_p representing the characteristic of the lateral stability of an automobile whose wheels are rigid in the lateral direction but elastic in the radial and tangential directions. Here η_2 is the ratio of the total lateral reaction, critical for the lateral axle stability, to the weight G_2 of the car, borne by the rear driving axle; γ_p the ratio of the total circumferential force P_p, generated and uniformly distributed over the driving wheels, to the weight G_2 of the car.

The lower straight part of the curve corresponds to the beginning of the slipping of the inner axle wheel; the upper, curved part to the beginning of the lateral sliding of the axle without the previous slipping of the inner wheel. The point of intersection of the two parts represents the simultaneous beginning of both the lateral sliding of the axle and the slipping of its inner wheel.

He then considers the motion of an automobile with elastic wheels and derives the equation $A_1\eta_2^2 + B_1\eta_2 + C_1 = 0$, A_1, B_1, C_1 containing γ_p and the parameters of the car. With this equation the corresponding characteristic is constructed for the case of simultaneous slipping and lateral sliding of the inner wheel of the driving rear axle. With the help of the correspondingly derived equations, he plots in the same figure a characteristic analogous to the one mentioned in the beginning but with this difference: now the upper part corresponds to the beginning of the lateral sliding of the driving axle in presence of slipping, and lateral sliding of the inner wheel; the lower part to infinite velocity of the same wheel. Finally, he obtains the contour of the characteristic of the stability of a car whose wheels have lateral elasticity.

Similarly may be constructed characteristics of the lateral stability of a car, for other, not uniform, distributions of the force P_p upon the wheels of the driving axle.

D. R. Mazkewich, USA

Gyroscopics, Governors, Servos

416. K. Stange, On the motion of a stable heavy symmetric top at small disturbances of the longitudinal oscillation (in German), Ingen.-Arch. 16, 343–356 (1948).

The motion of a top is studied, assuming that initially the axis of symmetry is vertical, but the angular momentum vector is near the vertical. For part I of this paper see Rev. 1, **1582**.

Courtesy of Mathematical Reviews P. Franklin, USA

417. Gino Arrighi, On the magnetic gyroscope (in Italian), Atti Accad. Naz. Lincei Mem. Cl. Sci. Fis. Mat. Nat. (8) 1, 195–204 (1947).

The "magnetic gyroscope," studied in this paper, is essentially a symmetric rigid body with a fixed point on the axis of symmetry

and subjected to forces due to two dipoles disposed as follows: one is fixed in the body along the axis of symmetry; the other is fixed in space in line with the fixed point of the body. The potential function due to these dipoles is expanded in a series of Legendre polynomials, but this seems to have little to do with the subsequent discussion of precession which is straightforward and leads to expected results. It is also to be expected (as verified by the author) that, when the poles of the spatially fixed dipole are very distant from the fixed point compared with the poles of the other dipole, the motion is like that of an ordinary gyroscope under the action of gravity.

Courtesy of Mathematical Reviews D. C. Lewis, USA

418. Victor Broida, Heat inertia in problems of automatic control of temperature, Instruments 22: Feb., 136–138 and 160–166; Mar., 222–224 and 254–264; Apr., 324–325 and 362–366; May, 406 and 450–458 (1949).

For a mechanical control system the differential equations must include terms representing the inertia of the system. In the corresponding case of the automatic temperature control of a heating unit, the equation requires terms to allow for heat lag or, as it is called in the terminology of the author, "heat inertia," since its effects are similar to that of inertia in a mechanical system. It can be argued that since inertia can only be connected with mass, the use of the phrase "heat inertia" is inappropriate, and "lag constant" could be used equally well.

The author formulates an equation and shows that it describes the performance of the various practical automatic temperature controls. The heating unit is replaced by an "equivalent" mass m of water, which, when heated to the same temperature as the heating unit, would behave in an identical manner on being subjected to the same disturbance. The value of m cannot be calculated for a given system but can be easily determined experimentally. It is shown that large values of m correspond to long times for temperature restabilization and vice versa. When the volumetric flow of fluid through the unit is instantaneously increased from q_1 to q_2, the heat supplied to the unit changes from Q_1 to $Q_1 + f(\phi,t)$, where $f(\phi,t)$ is determined by the characteristics of the temperature controller. A heat balance gives

$$cq_2(\theta - \theta_0) = -md\theta + Q_1 + f(\phi,t),$$

where ρ and c are the density and specific heat of the fluid respectively. This equation is used to determine the characteristics of a number of typical temperature-control systems.

G. M. Lilley, England

419. M. P. Alméras, A method for the improvement of the regulation characteristics of hydroelectric units (in French), Houille blanche 4, 38–43 (Jan.–Feb. 1949).

This paper discusses three possibilities of improving control promptness in modern hydroelectric units:

1. Addition of a manometric detector picking up the difference between the actual and the average pressure.
2. Addition of a device picking up the second derivative of the speed.

Both should be installed in connection with a governor responding to speed and acceleration.

3. Design of a governor with temporary control in which the tachometer is replaced by a mixed system detecting speed and acceleration.

From an analysis of the differential equations of the three systems it appears that they are effective and do not impair the stability.

No description is given of the mechanisms, although it is said that patents on the three procedures have been taken.

Wilhelm Spannhake, USA

Vibrations, Balancing

(See also Revs. 396, 516, 578, 580, 581)

420. Philip Hartman and Aurel Wintner, A criterion for the non-degeneracy of the wave equation, Amer. J. Math. 71, 206–213 (Jan. 1949).

The authors prove the following theorem: If the real continuous function $f(t)$ for $t \to \infty$ satisfies the condition max $[f(t),0] = O(t^2)$, the differential equation $x'' + f(t)x = 0$ cannot have two linearly independent solutions, both satisfying ${}_0\int^\infty x^2dt < \infty$.

In an appendix it is shown that if $f(t) \leqslant g(t)$ and the equation $x'' + g(t)x = 0$ possesses a solution x with $\int^\infty x^2dt = \infty$, then $x'' + f(t)x = 0$ need not have such a solution.

R. Timman, Holland

421. J. LaSalle, Relaxation oscillations, Quart. app. Math. 7, 1–19 (Apr. 1949).

This paper concerns the differential equation

$$d^2x/d\tau^2 + \mu f(x)dx/d\tau + x = 0 \tag{1}$$

originally established by van der Pol for $f(x) = x^2 - 1$ and generalized later by others. The author studies the relaxation range of this equation when the parameter μ is a large number, and starts his argument from the equation

$$Ld^2q/dt'^2 + F(dq/dt') + q/C = 0 \tag{2}$$

encountered in physics. By introducing the new variables z and τ connected to t' and q by the relations $q = (LC)^{1/2}z$; $t' = (LC)^{1/2}\tau$, (2) is reduced to Rayleigh's equation:

$$d^2z/d\tau^2 + \mu F(dz/d\tau) + z = 0; \ \mu = (C/L)^{1/2} \tag{3}$$

and it is shown that (1) and (3) are equivalent to the following two sets of equations in the two phase planes (x,w) and (x,y):

$$dx/dt = w; \ dw/dt = -f(x)w - \epsilon^2x$$

$$dy/dt = \epsilon^2x; \ dx/dt = -F(x) - y = G(x) - y$$

where $\epsilon = 1/\mu = (L/C)^{1/2}$; $t = \mu\tau = t'/C$; $y = \mu z = q/L$; $x = dz/dt = dq/dt'$; $G(x) = -F(x)$ and $f(x) = F'(x)$.

By the method of isoclines the author shows the behavior of integral curves in these two phase planes and notes that in the (x,y) plane the isoclines do not depend on ϵ. The results obtained are shown to be consistent with those obtained by other methods.

The principal part of the paper is devoted to the determination of the "enclosure," i.e., of a bounded region of the (x,y) plane which contains a periodic solution (i.e., a closed integral curve). This part being purely mathematical cannot be abstracted. It is based on the application of the Bendixson theorem which states that if the integral curves either enter or leave a bounded region of the phase plane not containing any singularities, then there exists a periodic solution in that region. By means of a rather long and laborious argument the author shows that such a region can be constructed.

As the result of this analysis, the author shows that the relation between the period T of a relaxation oscillation and the parameter ϵ established previously by Liénard, viz.: $\epsilon^2T \to 3 - 2 \log 2$, is correct.

N. Minorsky, France

422. W. J. Duncan, Some related oscillation problems, Coll. Aero. Cranfield Rep. no. 27, 17 pp. (Apr. 1949).

This paper is concerned with the natural frequencies associated with linear systems without damping and subjected to prescribed boundary conditions. Two problems are considered. In the first, a method is presented whereby the natural frequencies of a system can be duplicated by a change in the constraints. Suppose the displacement in some pure mode of oscillation satisfies the differential equation $f(D)u - \omega^2 u = 0$, where ω is the angular frequency of oscillation and $f(D)$ stands for a linear differential operator in the spatial coordinate. Then $v = \phi(D)u$ is also a solution provided $F(D)$ and $\phi(D)$ commute; v will not in general have the same boundary conditions as u. Examples of the application of this technique are drawn from the torsional oscillations of shafts and the lateral vibration of beams. It is well to point out that all of the examples given could have been treated equally well by the principle of duality.

In the second part, the author shows how to derive an expression for the natural frequencies of a system if it is composed of simply connected subassemblies whose driving-point admittances are known. (Reviewer's note: The author uses the term admittance for the ratio of displacement to force, whereas conventional usage reserves this terminology for the ratio of velocity to force.) The desired relation is simply that the sum of the admittances at the junction point is zero.

Horace M. Trent, USA

423. N. N. Bautin, On L. J. Mandelstam's problem in the theory of clocks (in Russian), Doklady Akad. Nauk SSSR 65, 279–282 (March 1949).

Let τ be the period of a pendulum and $\lambda_1, \lambda_2, \ldots$ various physical parameters upon which it may depend. In 1944 Mandelstam proposed the problem of comparing the partials $\partial\tau/\partial\lambda_\nu$ for clocks of Galileo-Huyghens type and for pre-Galilean clocks (i.e. without pendulum or spring). He suggested that this would bring out clearly the role of pendulum or spring as period stabilizers. This problem is solved by the author for the ideal clock model of his previous note [Doklady Akad. Nauk 61, 17–20 (1948); see Rev. **1244,** Oct. 1949].

S. Lefschetz, USA

424. W. T. Rollo and L. G. Chambers, The vibrations of a pair of truncated cones placed base to base, Philos. Mag. (7) 38, 609–634 (1947).

Using the conventional one-dimensional beam vibration equation, the eigenfunctions and eigenvalues associated with the structure of the title are determined. The exact solutions of the equation are found in terms of Bessel functions and certain approximate solutions are also developed. The agreement is excellent.

G. F. Carrier, USA

425. H. Haener, Vibrations of a combination of two cantilever beams. Öst. Ingen.-Arch. 3, no. 1, 30–39 (Jan. 1949).

A method is presented for the vibration analysis of a cantilever beam supporting, at its free end, a second cantilever beam at right angles to the first. After obtaining an expression for the potential energy, differential equations are obtained in the conventional manner by calculus of variation. The interesting feature of the paper is that the nongeometric boundary conditions are not determined separately, but are the "natural boundary conditions" of the variational problem, and appear automatically in the process of variation.

Hans Bleich, USA

426. W. S. Hemp, On the natural frequencies of a reinforced circular cylinder, Coll. Aero. Cranfield Rep. no. 26, 8 pp. (Mar. 1949).

The author studies the effect of shear flexibility on the natural frequencies of a cantilever in the form of a circular cylindrical shell made up of a skin carried on closely spaced stringers and rigid rings. The shell deflection is defined by a lateral deflection $V(x)e^{ipt}$ and a longitudinal displacement $U(x)e^{ipt} \cos \theta$, where cylindrical coordinates x, r, θ are used. Only the normal stress xx and the shear stress $x\theta$ are assumed to act. The boundary conditions are taken to be the vanishing of the displacements U, V at the root end and the vanishing of normal stress and total shear force on the free end of the cantilever. A frequency equation is derived which reduces, when shear flexibility is neglected, to the usual equation for the cantilever. The frequencies given by these two equations are compared numerically for one set of parameter values and are found to be approximately equal at the fundamental; higher modes are found to have much lower frequencies when shear flexibility is taken into account.

Robert D. Specht, USA

427. A. N. Markov, Dynamic stability of anisotropic cylindrical shells (in Russian), Prikl. Mat. Mekh. 13, 145–150 (Mar. and Apr. 1949).

The author investigates the problem of dynamical stability of thin orthotropic cylindrical shells subjected to pulsating loads. The formulation of the problem is based on Love's approximate theory and assumes that the changes of curvature are adequately characterized by the displacement in the direction of the normal to the surface. The author's formulas for critical loads specialize to known results in the isotropic case.

I. S. Sokolnikoff, USA

428. G. Petrashen, On the Lamb problem for an elastic half space (in Russian), Doklady Akad. Nauk SSSR, 64, 649–652 (Feb. 1949).

A suddenly applied isolated force on the surface of an elastic half space produces a vibration. Its theory, indicated in the paper, uses a complex function, Fourier integrals and Rayleigh's equation. The longitudinal and lateral vibrations on the wave fronts are given in closed form or as series expansions.

Z. Bažant, Czechoslovakia

429. G. Petrashen, The two-dimensional problem of Lamb for an infinitely elastic layer, limited by parallel planes (in Russian), Doklady Akad. Nauk SSSR 64, 783–786 (Feb. 1949).

If one plane of an elastic layer is free of stresses, and on the other plane acts an isolated, suddenly applied force, elastic waves are produced; the solution of these waves can be represented by Fourier integrals. The integrands are represented by series and discussed. The wave has as one component the wave of Rayleigh whose damping is compared with the damping in the case of an elastic half space.

Z. Bažant, Czechoslovakia

430. G. Heinrich, Gyroscopic effect in flywheel oscillations (in German), Öst. Ingen.-Arch. 3, no. 1, 23–29 (Jan. 1949).

The oscillation of a rigid flywheel, rigidly fastened to and rotating with a flexible shaft, has been considered by Timoshenko. The present author treats the problem of a rigid shaft bearing a flywheel with flexible spokes and a rigid rim at which all the mass is concentrated. In an earlier work, assuming that the resulting flexing of the flywheel caused no perturbations in the steady rotation of the shaft, the author had derived a simple solution. In the present treatment this assumption is discarded as being too restrictive. Here he assumes that (1) construction is the idealized one noted above; (2) the principal moments of inertia about axes in the plane of the wheel are equal; and (3) only small oscillations are considered.

The problem is set up using the Eulerian angles and equations

The simplifications introduced by the assumptions are inserted, and by ingenious quadratures the solutions are obtained. In transforming to a uniformly rotating coordinate system in the rest-plane of the flywheel, it is found that oscillations about normal axes in the rest-plane have two different frequencies, one for each axis. However, by the proper choice of the uniform rotation rate of the new coordinate system, only one frequency is present; thus, these are the normal coordinates. This rotation rate depends on the ratio of the principal moments of inertia of the flywheel about the shaft axis and about one in the plane of the wheel.

It is to be hoped that the next step will be to treat this problem with the shaft no longer assumed rigid. Also, asymmetry of the flywheel can be introduced for the case of an odd number of spokes. Vincent Salmon, USA

431. Arnold Sommerfeld, Special solutions of problems on elastic eigenvibrations of a parallelepiped and a cube (in German), S.-B. Math.-Nat. Abt. Bayer. Akad. Wiss. 1945/46, 81–88 (1947).

A family of eigenfunctions associated with the oscillations of a rectangular parallelepiped of isotropic elastic material are demonstrated. They are valid when one or more of the ratios of edge lengths are rational and correspond to the case where all faces are force free. The oscillations are characterized by the fact that the displacement is a two-component vector and that the dilatation (i.e., the divergence of the displacement) vanishes identically.

G. F. Carrier, USA

432. S. S. Manson, Stress investigations in gas turbine discs and blades, Soc. auto. Engrs. quart. Trans. 3, 229–239 (Apr. 1949).

The paper deals with vibration frequencies and vibration stresses in turbine blades of a typical gas-turbine engine; the operating temperatures in a turbine disk and the stresses occurring from the temperature gradients in that disk and the rotating speed. Temperature and stress readings have been taken by means of thermocouples and strain gages (platinum-irridium alloy) mounted at some blades and at the disk. The results are given in a number of graphs.

It is shown that at normal cruising speed the centrifugal stresses in the blades are already so high, that there is little room left for stresses due to vibrations. Those caused by the finite number of guide vanes (48) proved to be critical. Also vibrations of the seventh order (frequency equal to seven times the rotating speed in revs/sec) are important. It is possible that they are excited by inequalities of mass flow through the 14 combustion chambers. The stresses in the disk due to the rotating speed and the temperature gradient have been estimated by means of calculations. At the rim, high compressive stresses can occur which may lead to plastic flow. This in turn causes high residual tensile stresses after operation, so that rim cracking may follow. Cooling of the inner region of the disk during operation increases the temperature gradients and hence the stresses. In spite of the increasing strength of the material the degree of safety may be offset.

Some means for avoiding rim cracking are mentioned, one of them consisting of abandoning welded-blade constructions.

J. G. Slotboom, Holland

433. H. G. Yates, Vibration diagnosis in marine geared turbines (Discussion), Trans. N. E. Coast Instn. Engrs. Shipb. 65, D37–D50 (Apr. 1949).

The discussion brings out some further examples of the techniques developed by the author. Some interesting contributions to the problem of tolerable amplitudes of vibration at different frequencies from the viewpoint of human comfort are presented.

F. E. Reed, USA

434. P. Draminsky, Vibration-crankshaft damping, Shipbuilder 56, 347–352 (Apr. 1949).

The author presents an analysis of experiments on the damping of a crankshaft for a normal single-cylinder oil engine. The principal conclusions are: (a) the hysteresis damping per cycle is proportional to the square of the maximum stress over a wide range; (b) bearing damping is proportional to the bearing clearance; (c) bearing length and oil grooves have little effect on damping; and (e) damping is proportional to the unbalanced mass. Albert I. Bellin, USA

Wave Motion, Impact

(See Revs. 461, 472)

Elasticity Theory

(See also Revs. 429, 452, 454)

435. V. V. Krechmer, On some problems of the theory of mechanical similarity (in Russian), Doklady Akad. Nauk SSSR 65, no. 4, 457–460 (Apr. 1, 1949).

P and M are homogeneous isotropic elastic bodies, geometrically similar and of the same material, the linear dimensions of P and M being in the ratio $\lambda:1$ = constant. Let further the vectors of elastic displacements at corresponding points of P and M be denoted by μ_p and μ_m. If the forces acting on P and M are only surface forces, which are such that at corresponding points the condition (1) $\mu_p = \mu_m$ is satisfied, then the Navier equations of elastic equilibrium of P and M are the same with corresponding boundary conditions. In this case the stress tensors and deformation tensors at corresponding points of P and M are equivalent. If on the contrary the forces acting are body forces, for example, those due to gravity, i.e., if the states of stress of P and M are produced by gravitational forces and $\lambda \neq 1$, then the condition (1) is not satisfied, and therefore in this case the stress tensors and deformation tensors at corresponding points of P and M are not equivalent.

There follows a description of a method by means of which by introduction of artificial inertia forces, especially of centrifugal inertia forces, the statical similarity of P and M can be realized at least approximately [cf. Phillips, C. R. Acad. Sci. Paris 68, 75–79 (1869); Kirpichev, *Resistance of Materials*, part 2, 1922]. By modeling with the aid of the centrifugal adjustment it is, however, impossible to realize either the dynamical or the relative dynamical similarity, i.e., when the vectorial relations between the corresponding quantities are not satisfied, but only the scalar relations, referred to different coordinate systems. It is shown by an example that in special cases it might be useful to apply the criterion of partial relative similarity, based on the consideration not only of the kinematical and dynamical elements, but also of the elements determined by elastic forces, which are produced by suddenly applied loads. It is supposed that the loads are geometrically similar and are made of the same material and that the elastic deformations are propagating through the bodies instantaneously. The relations obtained are analogous to that given by the criterion of similarity of Cauchy.

Courtesy of Mathematical Reviews E. Leimanis, Canada

436. P. Lardy, The exact solution of the problem of the oblique plate (in German), Schweiz. Bauztg. 67, 207–209 (Apr. 9, 1949).

A closed solution is indicated (although no numerical examples are given) for a skewed plate in the shape of a parallelogram with simple supports on all four sides. The equation $\nabla^4\phi = p/N$ is

expressed in oblique coordinates u and v and the general solution is found in terms of functions of the characteristic argument $\alpha u + \beta v$. The satisfaction of the boundary conditions $\phi = 0$ and $\nabla^2\phi = 0$ involves the development into Fourier series, and the solution of several systems of infinitely many equations in the Fourier coefficients. E. F. Masur, USA

437. H. Reissner and M. Morduchow, Reinforced circular cutouts in plane sheets, Nat. adv. Comm. Aero. tech. Note no. 1852, 60 pp. (Apr. 1949).

A study is made for designing the reinforcement of a cutout in a plane sheet in such a way that it be as nearly as possible equivalent to the cutout part of the structure. First, general formulas are developed for the circumferential distribution of the cross-sectional moment of inertia and the area of a circular reinforcement required for the sheet with cutout to have the same stresses as the original sheet without cutout. Fourier expansion solution of the biharmonic equation for the Airy's function in polar coordinates is used to obtain stresses in the infinite sheet with a circular cutout. Thus the displacements are obtained from Hooke's law. Equilibrium considerations of a circular ring under normal and shear stresses in the sheet acting between the contacting surfaces of the ring and infinite sheet, and equating the displacement of the ring and sheet, lead to the area and moment of inertia distribution. Formulas obtained in this way are applied to the following cases: hydrostatic stress, pure shear, uniaxial tension, and pure bending. In the cases of uniaxial tension and pure bending it is shown that constraint stresses are unavoidable. Formulas are also developed for the stress distribution in a large plane sheet with a circular cutout reinforced by a ring of a given uniform cross section, not necessarily equivalent to the portion of the sheet removed. The report also contains test results.

A. Cemal Eringen, USA

438. A. E. Green, On Boussinesq's problem and penny-shaped cracks, Proc. Camb. phil. Soc. 45, 251–257 (1949).

Using a solution of an integral equation developed by Copson in connection with the problem of the electrified disk [Proc. Edinburgh Math. Soc. 8, no. 2, 14–19 (1947)], the author solves a number of "crack" and "punch" problems in elasticity. Most of these problems have been worked out by the solution of other types of integral equations. [See, for example, Harding and Snedden, same Proc. 41, 16–26 (1945).]

Courtesy of Mathematical Reviews A. E. Heins, USA

439. T. J. Willmore, The distribution of stress in the neighborhood of a crack, Quart. J. Mech. appl. Math. 2, 53–63 (Mar. 1949).

Using complex variable methods, an analysis is given for the distribution of stress in the neighborhood of a two-dimensional Griffith crack. First, a solution is developed for an isotropic elastic material when the pressure varies along the crack. The analysis is extended to the problem of a crack in an aeolotropic material with two directions of symmetry, and the critical pressure which will produce rupture is determined for the case of uniform normal pressure along the crack.

A solution is given also for the stress distribution in the neighborhood of two equal collinear cracks in an isotropic material, when a uniform pressure acts normally along each crack. It is shown that this problem is equivalent to the hydrodynamical problem of the uniform normal motion of two equal collinear flat plates through a fluid at rest at infinity. The solution is obtained in terms of elliptic functions and integrals, and calculated values of the maximum shearing stress are given for a particular case. It is found that the influence of one crack on the other is very small provided that the distance between the cracks exceeds the length of each crack. Dana Young, USA

440. M. A. Sadowsky and E. Sternberg, Stress concentration around a triaxial ellipsoidal cavity, J. appl. Mech. 16, 149–157 (June 1949).

This paper presents a solution of the equations of elasticity which has the following attributes: it is three-dimensional; does not involve an axis of symmetry; is an exact solution in closed form; and represents a technical problem of importance. Almost any one, and certainly any two, of these attributes would be sufficient to give a solution a position of prominence among the great number which have been found since the formulation of the general theory in 1821. When a solution is discovered which has all the above qualities, it is an unusual event.

The authors consider an ellipsoidal cavity in an infinite, isotropic, elastic body. At infinity the state of stress is homogeneous, with the directions of the principal stresses parallel to the principal axes of the cavity. The stress system required to annul the tractions across the cavity surface due to the homogeneous field is written in terms of four of Boussinesq's harmonic displacement potentials. The latter are then expressed as ellipsoidal harmonics composed of products of Lamé functions. The essence of the feat performed by the authors is the selection of the appropriate finite number of Lamé functions. They first eliminate functions leading to stress fields which do not vanish at infinity and, for the rest, they select those which will lead to tractions, across the surface of the cavity, having the same periodicity and symmetry as the tractions arising from the uniform field. They find that there are only six harmonic combinations of Lamé functions which satisfy the requirements and only five of these are independent, so that there are five arbitrary constants in the formulas for the components of stress. Application of the boundary conditions leads to sixteen linear equations in the five constants but, fortunately, the sixteen reduce to five independent equations. Accordingly the complete solution is expressible in terms of the homogeneous stress and the five harmonic functions. Numerical computations were made for the case in which the homogeneous field is a simple tension parallel to the smallest axis of the ellipsoid, and very complete data in the form curves are given for stress concentration factors at the most interesting points. R. D. Mindlin, USA

Experimental Stress Analysis

441. H. T. Jessop, The determination of the separate stresses in three-dimensional stress investigations by the frozen stress method, J. sci. Instrum. 26, 27–31 (Jan. 1949).

A method is proposed for separating principal stresses by stepwise integration along lines of principal stress in a principal plane. It is applicable with the frozen stress technique for a principal slice which is thin enough for the isoclinics and isochromatics to give correct values for the middle plane. Oblique incidence is suggested for determination of the principal stress differences at each point.

An equation of equilibrium along the P line of arc length S is derived: $\partial P/\partial S + (P-Q)/\rho_1 + (R-P)\cos\alpha/\rho_2 = 0$, where P and Q are the principal stresses in the plane of symmetry, R is the third principal stress, ρ_1 is the radius of curvature of the Q line, ρ_2 of the R line, and α is the angle between the x-axis and the principal normal to the R line. Except for special cases α_2 and ρ_2 cannot be determined so that $\cos\alpha/\rho_2$ is replaced by $\partial\psi/\partial S_2$, the rate of change of the P-line direction along the normal to the slice. Its determination requires cutting slices perpendicular to the principal slice and tangent to the P line.

The lateral strain method is said to be "obviously inapplicable since we have no means of measuring the strains." However, Kuske has shown that reheating the slice and thus releasing the frozen stresses provides an excellent means of obtaining lateral strains and thus separating principal stresses.

D. C. Drucker, USA

442. **Arthur C. Ruge, Bonded resistance wire gages for strain measurements,** Prod. Engng. 20, 116–117 (Jan. 1949).

Selection of the proper type bonded resistance wire strain gage is briefly discussed, consideration being given to the stress condition, gage length, gage size, and gage resistance. Details of construction and principal dimensions are clearly pictured.

Louis F. Coffin, Jr., USA

443. **E. A. Owen, A high-temperature X-ray camera for use with plate specimens,** J. sci. Instrum. 26, 114–117 (Apr. 1949).

This X-ray diffraction camera for use at temperatures up to 1000 C is arranged so that the specimen may be rotated in its own plane while it is oscillated through small angles about a vertical axis in the specimen plane. These simultaneous motions are necessary to avoid spottiness in diffraction patterns from metals with coarse grains or preferred orientation. In this design, the specimen chamber with provision for evacuation and heating is entirely separated from the film cassette and collimating system. This arrangement considerably simplifies the drive mechanism and permits convenient film loading and unloading without disturbing the vacuum or temperature in the specimen chamber. The entire specimen chamber is oscillated which obviates the need for transmitting this motion through the chamber wall by means of a vacuum seal. The zero point of oscillation can be changed to provide various mean glancing angles. The specimen is rotated by a magnetic drive through the specimen chamber wall; a small motor-driven bar magnet on the outside induces rotation of a similar magnet within the chamber. The specimen temperature is measured by simultaneously exposing a standard material of known thermal expansion coefficient.

This camera is especially suited to the study of high-temperature phase changes, and to determinations of thermal expansion coefficients; also, it can be used to determine the effects of annealing on internal stresses in metals. Maxwell Gensamer, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 461, 471, 500)

444. **Karel Válek, Elastic rigidity of continuous structural members,** Techn. Obzor 57, no. 3, 41–48 (March 1949).

The author refers to a previous paper [Techn. Obzor, 101–109 (1944)], and investigates structural members of various stiffness, and having constant and variable moments of inertia (haunched beams and columns). Tables were calculated for continuous beams of equal symmetric spans having three typical distances of conjugate points from the adjacent supports, u or v = 0.25, 0.33 and 0.40. Similar tables were calculated for continuous beams with symmetric spans, however, with increasing stiffness and with variable stiffness. As further examples, continuous beams on elastic supports, central hexagonal section of a grain elevator and a central rectangular unit of a structural network, are discussed.

J. J. Polivka, USA

445. **E. Panayiotounakou, Statical investigation of curved beams in space** (in Greek), Technika Chronika 26, nos. 295–296, 25–34 (Jan.–Feb. 1949).

This paper investigates the problem of spatial beams in which the line joining the centers of gravity of their cross sections is a continuous twisted curve of double curvature. The solution to this problem has been set up so as to make it applicable to any continuous beam in space, and is obtained by expressing the centroidal curve in terms of a general parameter u. Introducing the parameter s (arc length) gives a special case of the general solution in terms of u.

In setting up the differential equations for the stresses and the strains for a solid element of arc, ds, use is made of the theory developed by A. Roussopoulos at the National Metsovo Polytechnic Institute, Athens, Greece, on curved beams in space, and of his as yet unpublished article. Dimitri Kececioglu, USA

446. **E. Panayiotounakou, Transversely loaded circular beams** (in Greek), Technika Chronika 26, no. 298, 117–134 (April 1949).

This study presents new statical methods of solving the problem of a plane beam with a circular axis and loaded normally to the beam plane. The methods are based on the basic assumption that plane sections remain plane after deformation. It is pointed out that this assumption is valid only for beams of circular cross section or of section not far remote from circular. This is due to the appearance of a torsional moment in beams having different cross sections than the above. This moment invalidates the assumption.

In calculating the forces in statically indeterminate systems, the author neglects the effect of the shearing forces on the total work of deformation. He proves that the latter is negligible as compared to the effects of the bending and twisting moments in the theoretical analysis he presents. Dimitri Kececioglu, USA

447. **Sverre Sandberg and Julius Lindblom, Supports for power transmission lines** (in Swedish), Tekn. Tidskr. 79, 275–278 (Apr. 9, 1949).

The essential part of the paper is a description and illustration of a new supporting member between the cable and the insulator. It is bent along a circular curve and the bending moments in the power line are thereby reduced. The more theoretical part of the paper is of no great value. A. R. Holm, USA

448. **Th. Whyss, Effect of secondary bending and internal compression on the lifetime of stranded wire ropes with hemp core** (in German), Bauztg. 67: 193–198 (Apr. 2); 212–215 (Apr. 9); 225–228 (Apr. 16) (1949).

The paper forms part of a more extensive publication, which will be issued as report no. 166 of the Swiss Federal Materials Testing Institute (E.M.P.A.). It consists of three sections, the first of which deals with the calculation of secondary bending stresses in stranded wire ropes. It is shown that these stresses may be of considerable magnitude, which yields an explanation of the results of fatigue tests made by Woernle and Herbst.

The second section gives an analysis of the loads and stresses between the component parts of the rope and between rope and pulley, caused by axial tension of and by transverse forces on the rope. In addition, the results of transverse loading tests in the plastic region and of bending and torsion fatigue tests with single wires are communicated.

In the third section the formulas are applied for ropes, the dimensions of which have been determined using a design criterion given by Drucker and Tachau [J. appl. Mech. 12, A33–38 (March 1945)]. The stresses determined in this way are in good agreement with an empirical formula, based on tests carried out at E.M.P.A. In a final section, the important conclusions of this investigation have been summarized. F. J. Plantema, Holland

Plates, Disks, Shells, Membranes

(See also Revs. 426, 427, 436, 455, 458, 495, 496, 497, 580, 581)

449. F. Bauer, Rectangular plate loaded on its free edge (in German), Öst. Ingen.-Arch. 3, no. 1, 1–8 (Jan. 1949).

This paper presents a solution for the bending of a rectangular plate simply supported along two opposite sides. A third edge is either simply supported or fixed, and the remaining side is loaded either with shears or moments whose distributions are given by a cosine wave. The author shows that the solution may be given in closed form for each case and presents the solutions as formulas and curves. Paul F. Chenea, USA

450. E. H. Mann, Shearing displacement of a rectangular plate, Proc. Camb. phil. Soc. 45, 258–262 (Apr. 1949).

The longer sides of a rectangular plate are clamped to two rigid bodies, which undergo a relative displacement parallel to these edges. The short sides are free of stress. This plane stress problem is solved by assuming the displacements u, v (parallel to the long and short sides, resp.) in the form

$$u = B_0 y + \Sigma A_n \sin ny \cdot f_1(x) + \Sigma B_n \cos nx \cdot g_1(y),$$

$$v = A_0 x + \Sigma A_n \cos ny \cdot f_2(x) + \Sigma B_n \sin nx \cdot g_2(y).$$

They satisfy one boundary condition on each side, the other one being used to determine the A and B. This is done by a rapidly converging iteration, and it is claimed that also the resulting series converge much better than those used by C. E. Inglis in an earlier paper [Proc. Roy. Soc. A 103, p. 598 (1923)]. W. Flügge, USA

451. K. H. Boller, Preliminary report on the strength of flat sandwich plates in edgewise compression, For. Prod. Lab. Rep. no. 1561, 29 pp. (1947).

Six facing materials and ten core materials were combined to make 169 sandwich constructions of three sizes. The paper reports the results of tests run to determine the properties of the facing and core materials, and the properties of the sandwich constructions.

The sandwich constructions were tested under edgewise and flatwise compression and flatwise tension. The different types of failure were observed and classified. A. J. Durelli, USA

452. S. A. Ambartsumyan, On the calculation of shallow shells (in Russian), Prikl. Mat. Mekh. 11, 527–532 (1947).

The author investigates stresses in a shallow rectangular spherical shell with nonvanishing Gaussian curvature [see Rev. 1, **425** (1948)] which is freely supported along the edge and subjected to normal loading. The computations are formal and include expressions for the transverse deflection of the shell. Numerical results are obtained for the case of a uniform normal load. I. S. Sokolnikoff, USA

453. F. B. Hildebrand, E. Reissner, and G. B. Thomas, Notes on the foundations of the theory of small displacements of orthotropic shells, Nat. adv. Comm. Aero. tech. Note no. 1833, 59 pp. (Mar. 1949).

It is well known that the solutions given in the theory of the bending of bars, plates and shells are no exact solutions of three-dimensional stress problems, but approximations, which are the better, the thinner the body in question. The fundamental relations (equilibrium of element, elastic law, kinematic relations) from which they are derived, are therefore not the last word on the subject, but capable of improvement.

The authors have studied this problem for shells. They give a critical survey of different formulations of the fundamental equations (Love, Basset, Trefftz, Synge and Chien), applying the ideas of these authors to a shell of special anisotropy. This anisotropy is so chosen, that an infinite value of one of the moduli is equivalent to the usual neglect of deformation due to transversal shear or of the elastic change of wall-thickness.

This survey of existing theories is followed by the authors' own contribution to the problem. They go beyond the limit where the state of stress can still be described by force resultants and moments, and introduce new stress integrals of higher order representing, e.g., the nonlinear distribution of bending strain over the thickness of the shell. To this degree of approximation it is possible to split the Kirchhoff boundary forces into their component parts and to establish independent boundary conditions for each of them. Thus also such problems may be solved, where the width of a boundary zone of disturbance is of the order of magnitude of the shell-thickness while usual shell theory is confined to cases where this width is the geometric means of thickness and radius of curvature. W. Flügge, USA

454. I. Fytos, Hyperbolic paraboloid shells (in Greek), Technika Chronika 26, nos. 295–296, 35–44 (Jan.–Feb. 1949).

In this paper the expressions for the stress distribution in a hyperbolic paraboloid shell are derived. The existence of bending moments is excluded. It is pointed out that shells of double curvature have the advantage over cylindrical shells in that they deform much less under load and in that slight deformations do not alter the stress distribution appreciably. One such shell is the hyperbolic paraboloid, which possesses the following significant properties:

(a) If at the edges of the hyperbolic paraboloid there act forces in the direction of the generator, these forces are transferred along the generator and the body of the shell without altering the stress distribution, and are simply added to the existing stresses. This facilitates the determination of the stress distribution along the supports of the shell.

(b) Hyperbolic paraboloid shells form a surface of equal resistance to normal and uniformly distributed loads, i.e., at any point in the shell surface the induced stresses have the same value.

(c) The hyperbolic paraboloid is a developable surface. This makes its construction easy.

Nine different types of hyperbolic paraboloid surfaces and their supports are described, and one numerical example is worked out. Dimitri Kececioglu, USA

Buckling Problems

(See also Rev. 481)

455. D. Yu. Panov and V. I. Feodos'ev, Letter to the editor (in Russian), Prikl. Mat. Mekh. 13, 116 (1949).

Attention is called to a missing term in equation (3.7) of the author's paper on stability of shallow shells [same source, 12, 389–406 (1948); Rev. **181** (Feb. 1949)]. The inclusion of this term does not alter the method of solution but changes the values of the critical loads. I. S. Sokolnikoff, USA

456. Paul Seide and Manuel Stein, Compressive buckling of simply supported plates with longitudinal stiffeners, Nat. adv. Comm. Aero. tech. Note no. 1825, 23 pp. (Mar. 1949).

A solution using the Rayleigh-Ritz energy method is presented for the uniaxial compressive buckling of simply-supported plates with identical equally spaced longitudinal stiffeners. It is assumed that the stiffeners have zero torsional stiffness, but the

bending strain energy associated with the buckling of the stiffeners is included in the total internal energy. Charts are drawn, based upon the stability criterion developed in the analytical study, for the buckling of plates with one, two, three and an infinite number of stringers. The charts are particularly useful to designers and stress analysts since, in addition to the determination of the buckling stress as a function of plate and stiffener rigidity and panel dimensions, they enable also the direct selection of the minimum stiffener rigidity which precludes buckling of the stiffeners. John E. Goldberg, USA

457. A. Schubert, The buckling load of thin circular ring plates under uniform pressure on the inner and outer border (in German), Z. angew. Math. Mech. 25/27, 123–124 (1947).

The stability of a thin elastic plate with concentric circular boundaries under uniform radial pressure is investigated. The displacement function is found in terms of Bessel functions and the critical load is associated with the roots of a transcendental equation. Some numerical results are computed.

G. F. Carrier, USA

458. Paul Seide and Elbridge Z. Stowell, Elastic and plastic buckling of simply supported Metalite type sandwich plates in compression, Nat. adv. Comm. Aero. tech. Note no. 1822, 24 pp. (Feb. 1949).

The theory developed by Charles Libove and S. B. Batdorf [A general small-deflection theory for flat sandwich plates, same source, no. 1526 (1948)] is applied to the problem of compressive buckling of simply-supported Metalite-type sandwich plates and is also extended to the plastic range. It is assumed in the elastic range that the faces and core are isotropic, the applied loads are carried only by the faces, vertical shear forces are carried only by the core, and the faces are very thin compared with the core. The equations for the plastic range are based on the additional hypotheses that the core is elastic and no unloading occurs during buckling. Buckling loads for simply-supported, finite and infinitely long plates are calculated. Comparison with experiment shows fair agreement and those discrepancies which do occur may be traced to scattering and to the use of an average stress-strain curve for the face material. G. H. Handelman, USA

Joints and Joining Methods

(See also Rev. 482)

459. T. D. Tuft, Tensile tests of small-scale welded joints, Weld. Res. Suppl. 14, 41–48 (Jan. 1949).

Comparative tension tests are reported for Everdur brazing and mild-steel welded joints in black-iron sheet metal 0.024 in. to 0.10 in. thick. Transverse and longitudinal butt joints and longitudinal tee and angle specimens were tested to study these methods of joining thin metal for making $^1/_{10}$-scale model ship structures to be subjected to destructive tests by underwater explosion. Although the Everdur brazing does not develop the full strength or ductility of the metal, it is considered by the author to be satisfactory for material too thin for steel welding.

Henry A. Lepper, USA

460. C. E. Sims and H. M. Banta, Development of weldable high-strength steels, Weld. Res. Suppl. 14, 178–192 (Apr. 1949).

In this and previous investigations it was shown that the occurrence of cracks in the heat-affected zone of welds in 1-in. plate was not caused by high hardness and thermal stresses alone. Underbead cracking is thought to be caused by the delayed release of hydrogen. By using electrodes of low hydrogen content, the cracking of an otherwise crack-sensitive steel was practically eliminated. The use of such electrodes, however, is not yet acceptable for all types of work, and other means of crack prevention are sought. Tests on a large variety of steels showed that increases in carbon and manganese content caused an increase in the underbead cracking. Additions of vanadium and molybdenum increased the strength without increasing the cracking. The use of small amounts of aluminum was very detrimental, but larger amounts could be used safely. The cracking could be eliminated by keeping the carbon and manganese low, but the impact strength suffered. The impact strength could be increased by quenching and tempering the plate before welding. High strength and notch-bar toughness with low underbead cracking were obtained from quenched and tempered plate made of steel in which the carbon and manganese were limited. The heat treatment did not lower the joint efficiency of the plate.

Evan A. Davis, USA

461. W. J. Krefeld et al., An investigation of beams with butt-welded splices under impact, Welding Res. Suppl. 12, 372–432 (July 1947).

Part I reports tests of 18 standard H-beams, 16 in. deep, center loaded on 12-ft spans. Resistance of beams with and without standard butt welds was compared under static loads and under increment drop tests made at room and low temperatures. The welds had little effect on general behavior at room temperature but caused brittle fracture under impact at −40 F, while similar unwelded beams withstood greater impacts and lower temperatures without fracture.

In part II, M. G. Salvadori extends previous theories of Timoshenko, Lennertz, Zener and Feshbach, and Lee to cover more general relations between load and local deformation than the usual relation given by the Hertz sphere-contact theory. Some comparisons of this theory with the tests are made. Ed.

Structures

(See also Revs. 437, 444, 447, 500, 563)

462. J. Aggelopoulos, The computation of moments and stresses in elastic beam-girder type structures (in Greek), Technika Chronika 26, no. 298, 135 (Apr. 1949).

This is the abstract of a thesis presenting a new method of calculating the moments in the intermediate beams and the pressures on the main girders; in addition to the loads, the deflections at the locations where the beams bear on the girders are considered.

It is shown that the above exact method should be used in cases where the arrangement of beams and girders is such that a substantially nonuniform distribution of deflections is created at the various supports. It is also pointed out that the simplified method, where the deflections are neglected, should be used only when the differences in the elastic deflections at the various supports are small. An example is worked out where, though the simplified method gives permissible stresses, the exact method gives stresses that exceed the allowable ones.

Two methods are used to solve the girder-beam problem: first, by setting up a system of equations of elasticity; second, by successive approximations. The first method presents computational difficulties and is of theoretical significance only (except in a few simple cases). The second method, as developed in this thesis in an original manner, gives a simple and systematic way of solving the girder-beam type structure of any arrangement. It can be easily applied even to the simple cases. The required computations are easy and concise, and make it possible to com-

pute moments and stresses to any desired accuracy. It is pointed out that this method can also be applied when there is an interaction between girder and beam at the locations where they bear on each other. The paper is to be continued.

Dimitri Kececioglu, USA

463. A. J. Ashdown, Multiple-span prismatic thin-slab structures, Concr. constr. Engng. 64, 3–7 (Jan. 1949).

This paper is the third of a series in the same journal in which the author has developed methods for determining stresses in prismatic thin-slab structures subjected to loads at the slab intersections. The first paper (October 1948) outlined the basic theory and gave a solution for a prismatic thin-slab structure of one span with the edge tie members supported only at the ends. The second paper (November 1948) presented solutions in which (1) the edge tie members were supported along their length but free to slide, and (2) one edge tie member was supported and the other unsupported. The present paper extends the theory to multiple-span prismatic thin-slab structures.

The formulas presented are approximate because in assuming the loads concentrated at the intersections of the slabs no account is taken of the negative transverse bending moments and the relative deformation of the junctions. The effect of shear lag has also been neglected.

A similar treatment of prismatic thin-slab structures has been given by G. Winter and M. Pei [J. Amer. Concr. Inst. 18, p. 505 (Jan. 1947); see Rev. **80**, Jan. 1948]. The results of the present paper, obtained independently, agree with the solution of Winter and Pei except in the assumptions involving the distribution of the shearing stresses.

Karl Arnstein, USA

464. V. Koloušek and J. Blažek, Continuous arches on elastically yielding piers. Theoretical computation compared with results of experimental analysis (in Czech), Techn. Obzor 57, no. 2, 21–28 (Feb. 1949).

If continuous arches are supported by piers resting on elastically yielding soil, the theoretical analysis of internal forces becomes very intricate and cumbersome. It is shown that a relatively quick solution may be obtained by tests on small-size models, much as in the Beggs deformeter method, and that the results corroborate the theoretical analysis. A thorough analysis is presented including influence lines of bending moments, direct stress and shear. A simplified method is proposed which yields satisfactory results as compared with both theoretical and experimental analyses.

J. J. Polivka, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 504, 506)

465. A. I. Gubanov, Mechanics of elastoviscoplastic bodies: I, II (in Russian), Zh. tekh. Fiz. 19, 34–61 (Jan. 1949).

The paper produces a new penetrating critical survey of rheological models of real bodies whose proprieties deviate from perfect elasticity as well as from perfect fluidity. Those models are lucidly visualized by diagrams. The author points out that such a survey was already made by M. Reiner in his *Ten lectures on theoretical rheology* (published 1947 in Russian translation), but because "of certain errors and the omission of Soviet papers it requires a supplement."

The author proceeds to present Maxwell's model (1867) in terms of the deviators of strain tensors and stress tensors, viscosity, and relaxation-time, pointing to the new foundations of this model in I. Frenkel's *Kinetic theory of liquids* (1946). It is applicable to hot glass, plastics, etc. He handles in the same way Kelvin's model which connects elasticity with viscosity and is applicable, for instance, in soil mechanics and other fields.

Passing to models consisting of more than two elements the author deals with two Frenkel models and with one of his own. He calls the corresponding medium "Frenkel's liquid." He mentions further Bingham's plastic body with strain-hardening, and finally proposes the most general mathematical diagram of elasto-viscoplastic bodies including the first and second time-derivatives of quantities which characterize strain and stress. At the end of part I the author derives differential equations of motion of such bodies in analytic and vector form, and points to known difficulties in distinguishing elastic bodies from the elastoplastic ones. Part II is devoted to the solution of differential equations given in part I in a particularly simple case of pure shearing strain. Applying operational methods the author finds first the solution in the case of the given displacements and discusses it in detail for each type of model. He examines in the same way the case of given forces, and finally the case of impact, deriving copious formulas which he uses for the case of the torsion of a round cylinder and the stretching of a prism of elastoviscoplastic material.

M. T. Huber, Poland

466. H. J. Greenberg, On the variational principles of plasticity, Brown University Rep. no. Al-S4, 112 pp. (Mar. 1949).

The monograph provides a unified presentation of variational principles of plasticity for materials without time and temperature effects. Both deformation, or total, and flow, or incremental stress-strain relations are studied. Variational principles for deformation theories apply definitely only if loading always occurs everywhere—a very serious restriction.

Among the many interesting theorems discussed are:

1. A solution for the strains for any deformation theory [of the form $\sigma_{ij} = \sigma_{ij}(\epsilon_{pq})$ such that $\partial\sigma_{ij}/\partial\epsilon_{pq} = \partial\sigma_{pq}/\partial\epsilon_{ij}$] renders the "potential energy" φ an extremum with respect to admissible strain variations. Here $\varphi = {}_V\int {}_0\int^{\epsilon_{ij}} \sigma_{pq} d\epsilon_{pq} dV - {}_{S_2}\int T_i u_i dS_2$ where S_2 is the boundary area over which stress is specified. It is important to note that the potential energy is not rendered an extremum for the stress problem.

2. A solution of the stress problem for any deformation theory [of the form $\epsilon_{ij} = \epsilon_{ij}(\sigma_{pq})$ such that $\partial\epsilon_{ij}/\partial\sigma_{pq} = \partial\epsilon_{pq}/\partial\sigma_{ij}$] renders the "complementary energy," φ_c, an extremum with respect to admissible strain variations. $\varphi_c = {}_V\int {}_0\int^{\sigma_{ij}} \epsilon_{ij} d\sigma_{ij} dV - {}_{S_1}\int T_i u_i dS_1$ where S_1 is the boundary area over which displacement is specified.

3. A modified Haar-von Kármán principle is proved in which the elastic complementary energy is an absolute minimum. The first variation however is not zero.

The following two theorems established for flow or incremental theories are probably of greater future importance because the range of applicability of deformation theory is so strongly restricted.

A. Of all admissible systems of stress rates (or stress increments) $\dot\sigma_{ij}$ with corresponding strain rates (increments) $\dot\epsilon_{ij}$, given by $\dot\epsilon_{ij} = \dot\epsilon^e_{ij} + \dot\epsilon^p_{ij}$, $\dot\epsilon^p_{ij} = G\dot f \partial f/\partial\sigma_{ij}$, the true-stress-rate solution makes $\frac{1}{2}\int \dot\sigma_{ij}\dot\epsilon_{ij} dV - {}_{S_1}\int \dot T_i \dot u_i dS_1$ an absolute minimum. The loading function f is any function of stress, isotropic or anisotropic. Admissible means that equilibrium is satisfied and stress boundary conditions on S_2 are met. Superscript e refers to elastic, p to plastic.

The same theorem applies to perfectly plastic materials for which $\dot\epsilon^p_{ij} = \lambda\partial f/\partial\sigma_{ij}$ and $\dot\sigma_{ij}\dot\epsilon_{ij} = \dot\sigma_{ij}\dot\epsilon^e_{ij}$.

B. Of all admissible systems of strain rates with corresponding stress rates given by the inverse of the stress-strain relation, the true strain rate solution makes $\frac{1}{2}{}_V\int \dot\sigma_{ij}\dot\epsilon_{ij} dV - {}_{S_2}\int T_i u_i dS_2$ an absolute minimum.

The two theorems together furnish a means of getting upper and lower bounds. D. C. Drucker, USA

467. S. T. Kishkin and S. I. Ratner, Experimental check of the fundamental law of the plasticity theory (in Russian), Zh. tekh. Fiz. 19, 412–420 (Mar. 1949).

The object of the tests was to establish the relation between the yield stresses in shear (τ_y) and in tension (σ_y) for various materials. τ_y was defined as the shear stress at which the permanent set is 0.3% (which corresponds to 0.2% permanent set in tension) and was determined graphically. The ordinates of the stress-strain diagram were calculated according to the Nádái-Ludwig formula $\tau = (2\pi r^3)^{-1} (3M + \theta dM/d\theta)$ (the second term in the parentheses takes account of the strengthening effect of plastic deformation). The yield stress in tension σ_y was defined as the stress at which the permanent set is 0.2%. The ratio τ_y/σ_y which according to Saint Venant has a value of 0.5, whereas according to the theory of Huber-Mises a value of 0.577, was found to vary in wide limits from 0.25 for magnesium alloys, to 0.74 for high quality heat-treated steel. For pure metals with tetragonal crystal lattice (copper, iron and aluminum) the ratio τ_y/σ_y is 0.48–0.49. For pure metals with hexagonal lattice the ratio is very low; for pure magnesium it has a value of 0.27. Annealed steel gives a value 0.5, whereas heat-treated steel can have a value as high as 0.7–0.8. The mean value for aluminum alloys is about 0.4, but some cast aluminum alloys give a value 0.67. The authors have also determined the value of the yield stress in torsion from the conventional formula $\tau_y' = 16M/\pi d^3$ which was found to be 20–30% higher than the correct value calculated from the Nádái-Ludwig formula.

M. M. Gololobov, Czechoslovakia

468. K. Goldsmith and J. Iball, The measurement of the rheological properties of visco-plastic substances, Proc. phys. Soc. Lond. Sect. B, 62, 251–256 (Apr. 1949).

The authors assume the rate of change of shear strain to be proportional to a power of the shear stress, as well as to a power of the time, and to be inversely proportional to a "firmness-intensity factor." By means of the obtained equations, these exponents and this factor are calculated from the observed relation between blade deflection and elapsed time.

P. P. Bijlaard, USA

469. Cesare Castiglia, Static effect of cyclic distorsions in viscous regime (in Italian), G. Gen. civ. 87, 132–137 (Mar. 1949).

The total displacement in viscoplastic flow is decomposed into parts, comprising elastic, viscous and a superimposed cyclic displacement. The viscous part is proportional to the stress and to a "specific viscosity," which is assumed to depend upon time, approaching asymptotically a limiting value according to an exponential law. Assuming the elastic part of the superimposed motion to be of sinusoidal form, a differential equation for stress as a function of time is obtained, the integration of which is carried out in analytic form. Application to stress variations in fire-brick materials is given. Folke K. G. Odqvist, Sweden

470. G. P. Zaĭtsev, The Hertz problem and the Brinell test (in Russian), Zh. tekh. Fiz. 19, 336–346 (Mar. 1949).

Assuming that there is no permanent indentation below a certain critical value of the load in the Brinell test, the author believes that this value can be obtained from the two equations represented by the Hertz formula and the empirical Meyer hardness formula; he assumes that both formulas hold simultaneously at the postulated critical load, and thus represent a system of two equations for the two unknowns, load and indentation diameter. He remarks that there is no reason for surprise at the resulting relationship between the elastic constants and the Meyer parameters because both elastic and plastic quantities are ultimately determined by the properties of the atoms and the structure of the crystal lattice. E. Orowan, England

471. W. Swida, The elastoplastic bending of curved bars (in German), Ingen.-Arch. 16, 357–372 (1948).

The author discussed first the elastoplastic flexure of a naturally curved bar with constant radius of curvature and rectangular cross section. It is found that even when the radius of curvature is of the same order of magnitude as the height of the cross section, the greatest radial stress is considerably smaller then the greatest circumferential stress. The author then proceeds to develop an approximate theory of elastoplastic bending of naturally curved bars in which the transverse (radial) stresses are neglected. It is worth noting that the bending moment which produces collapse according to this theory is independent of the radius of curvature of the bar. W. Prager, USA

472. V. S. Lensky, On the elastoplastic impact of a rod against a rigid obstacle (in Russian), Prikl. Mat. Mekh. 13, 165–170 (Mar.–Apr. 1949).

A cylindrical rod of elastoplastic material with a linear work-hardening law is considered to strike a rigid target normally. The resulting system of elastic and plastic longitudinal waves of plane stress is analyzed.

For sufficiently high velocities an elastic wave front followed by a plastic wave front emanates from the point of impact. The reflection of the elastic wave at the free end interacts with the initial plastic wave front, causing unloading. Repeated application of the momentum change conditions across wave fronts caused by repeated reflection permits complete analysis of the motion. The rod finally rebounds in elastic vibration having been subjected to plastic strain in a series of bands of constant strain magnitude adjacent to the impact end. The strain distribution and time of impact are obtained as a function of the impact velocity.

The author states that the analysis is valid only if the particle velocity can be neglected in comparison with the wave velocity. This restriction, however, is unnecessary if nominal stress and nominal strain are used in conjunction with Lagrange coordinates based on the unstrained bar. The bookkeeping of the wave interactions would be simplified by use of wave trajectories in the xt plane, as carried out by Lee and Tupper in treating the same problem with a more complex stress-strain relation (British Official Report, Armament Research Report, Theoretical Research Report 4/44, 1944) and by White for a similar problem [J. appl. Mech. 16, p. 39 (1949); see Rev. 2, **839**].

E. H. Lee, USA

473. R. Hill, The plastic yielding of notched bars under tension, Quart. J. Mech. appl. Math. 2, 64–71 (Mar. 1949).

The state of stress at the root of notched bars is analyzed when pronounced plastic flow occurs. Plane strain and constant plastic volume change are assumed. The analysis is similar to that of Van Iterson [*Plasticity in engineering*, Blackie, 1947; see Rev. 2, **47**] in that Hencky's slip-field equations are applied, together with a constant-yield-stress material following the maximum-shear theory. The maximum-constraint factor (ratio of average stress to yield stress in simple tension when extension occurs) is 2.57 for an infinitely sharp parallel-sided notch. The present analysis discriminates between deep and shallow notches and devotes con-

siderable attention to the mechanism involved when extension of the bar occurs. The problem of the indentation of a metal by a flat punch (identical with that of the infinitely sharp parallel-sided notch), as developed by Prandtl, is discussed with particular emphasis on the nature of plastic flow when considerable indentation occurs. The author points out that considerable error in the boundary of the plastically deformed region occurs in problems of nonsteady-state flow (as in the notch or punch) if one considers only the slip field. For a more exact study, it is necessary to introduce the Saint Venant equations relating strain rate to stress.
Louis F. Coffin, Jr.

474. V. V. Sokolovskii, Equations of plastic equilibrium for plane stress (in Russian), Prikl. Mat. Mekh. 13, 219–221 (Mar.–Apr. 1949).

The differential equations of the plastic field for plane stress can be either hyperbolic or elliptic. The author discusses the solution

$$\left.\begin{matrix}\sigma_x\\ \sigma_y\end{matrix}\right\} = \frac{\sigma_s}{\sqrt{3}}\,(\sqrt{3}\cos\omega \pm \sin\omega\cos 2\varphi),$$

$$\tau_{xy} = \frac{\sigma_s}{\sqrt{3}}\sin\omega\sin 2\varphi,$$

of the plasticity condition for plane stress (σ_s is the uniaxial yield stress); substitution in the equations of equilibrium and calculation of the characteristics show that the equations are hyperbolic if $3 - 4\cos^2\omega > 0$, and elliptic if $3 - 4\cos^2\omega < 0$. The author's result can be expressed by saying that the equations are hyperbolic if the yielding is due mainly to the difference between the principal stresses in the xy plane (namely, if the maximum value of τ_{xy} is not less than $^1/_3$ of the mean principal stress in the xy plane), and elliptic if yielding is due mainly to the differences between $\sigma_z = 0$ and the two other principal stresses (namely, if the mean principal stress in the xy plane is more than three times higher than the maximum of τ_{xy}). The two cases are discussed from the mathematical point of view.
E. Orowan, England

475. V. V. Sokolovskii, A plane problem of the plasticity theory on stress distribution around holes (in Russian), Prikl. Mat. Mekh. 13, 159–164 (Mar.–Apr. 1949).

A method of finding the stresses in the plastic zones in the neighborhood of a hole was given by S. A. Khristianovich for the case of plane strain, and by the present author for the case of plane stress. This method leads to boundary-value problems for a system of equations of the hyperbolic type, and the solution is found numerically. The present paper is devoted to another method for determining the stresses around a hole for plane stress or plane strain. This method uses trigonometric series.
W. Prager, USA

476. E. A. Johnson, The relaxation test in terms of creep and creep recovery, Metallurgia 39, 291–297 (Apr. 1949).

The author showed previously (B.E.A.I.R.A., J/E Committee, J/T144 and 145) that the stress relaxation of a low-carbon steel and a chrome-molybdenum steel cannot be interpreted, using normal creep-test data, either by the time-hardening or the strain-hardening theories of creep. He assumes this fact to be due to the influence of other factors among which the creep recovery associated with the reduction of stress might have a special importance. In the present paper he describes creep-recovery tests with the above materials at 445 and 485 C, respectively, made in such a way that after suitable intervals of relaxation the load was removed and the reduction in length measured as a function of the time. The total time of stress relaxation and creep recovery was about 10,000 min in each case. The creep recovery ΔR due to a relaxed stress decrement $\Delta\sigma$ was calculated by $\Delta R = R \cdot \Delta\sigma/\sigma$, where R is the creep recovery when the stress σ is removed. The corrections of the creep rates during the relaxation tests obtained in this way are only 7 and 11%, respectively, and are too small to bridge over the disagreement between the experimental and theoretical results referred to above.
Albert Kochendörfer, Germany

477. G. R. Wilms and W. A. Wood, Mechanism of creep in metals, J. Inst. Metals, 16, 693–706 (Apr. 1949).

Creep of metals contains two phenomenologically different components, "transient" and "quasi-viscous" creep, which recently have also been shown to be physically distinct. Definite information is gained from a correlated metallographic and X-ray study of the creep behavior of annealed tensile specimens of aluminum.

Deformation at room temperature is characterized by transient creep. The X-ray diagram shows transition from sharp discrete spots to continuous rings. The microscope shows grains becoming gradually traversed by slip lines. The spread of the initial X-ray spots signifies breakdown of the perfect grains into variously oriented elements. The continuity of the arcs indicates breakdown into very large number of small "crystallites." These together represent the mechanism of deformation by slip.

Slow deformation at elevated temperature is characterized by quasi-viscous creep. In the X-ray diagram, the sharp initial spots break up into small groups of equally sharp secondary spots (10 to 20 for 1 original spot) which show progressive scattering until distribution becomes fairly uniform. The microscope shows absence of slip bands but progressive thickening of grain boundaries. The original grains, though conserving their identity, after deformation subdivide into a coarse structure of cells, the deformation proceeding by relative motion of the cells.

Deformation at different rates at elevated temperature produces a composite structure in which cell formation was more pronounced the higher the temperature and the slower the rate of deformation.

Only slip deformation gives significant strain hardening, because only in slip are the grains fragmented into small elements.

Spontaneous transition may occur from one type of structure into the other: rapidly strained specimens held at the same elevated temperature did not develop a structure comparable with slowly strained specimens. Thus cell formation can be produced only when straining and heating are applied simultaneously. It is concluded that the cell formation represents a definite substructure in the grains, analogous to the substructure suggested by the slip lines.
B. Gross, Brazil

Failure, Mechanics of Solid State

(See also Revs. 448, 461, 473, 491, 495, 500, 585)

478. T. A. Kontorova and O. A. Timoshenko, Generalization of the statistical theory of strength for nonuniformly stressed bodies (in Russian), Zh. tekh. Fiz. 19, 355–370 (Mar. 1949).

In several earlier papers, Kontorova has treated the size effect in brittle fracture, by assuming a Gaussian distribution of strengths in an assembly of specimens of equal size, and a proportionality between the number of flaws and the volume of the specimen. (In reality, for vitreous materials, e.g., this number would be proportional to the surface area rather than to the volume; Kontorova's assumptions are more justified for materials like plaster of Paris.) The present paper extends the earlier statistical calculations to the bending of bars of rectangu-

lar, and the torsion of rods of circular cross section. The results show that, with the assumptions made, the mean fracture stress of specimens of given volume would be lower in tension than in bending or torsion. This conclusion agrees with earlier measurements on specimens of plaster of Paris.

E. Orowan, England

479. G. G. McDonald, The graphics of pulsating stresses, J. Instn. Engrs. Austral. 20, 195–196 (Dec. 1948).

The author describes a few types of fatigue-failure graphs corresponding to different experimental combinations of steady and pulsating normal and shear stresses caused in a notch-free cylindrical shaft by combinations of bending and twisting pulsating moments. Comparison is made with results obtained by using several theories of failure. C. Cattaneo, Italy

480. W. A. Wood, Crystallite theory of strength of metals, Inst. Metals 16, 571–594 (Mar. 1949).

It is considered that for a given metal there is a fundamental structural unit of grain size below which the grain cannot be broken down by further plastic deformation in the cold. The limiting crystallite size varies with the metal and has been determined for four body-centered cubic metals by a back-reflection X-ray technique which consisted of measuring the variation in line broadening, using radiation of two different wave lengths so that the effects of internal strain could be eliminated. The values obtained for iron, molybdenum, tantalum and tungsten were 3.0, 4.2, 2.4 and 2.9 $\times$ 10^{-6} cm respectively. It is suggested that the crystallite size determines the limit of the amount of deformation by slip, and on this basis the calculated tensile-strength values for the four metals are of the same order as those obtained experimentally. B. W. Mott, England

481. William Le Fevre, Jr., Torsional strength of steel tubing as affected by length, Prod. Engng. 20, 133–136 (Mar. 1949).

The effect of length on the average torque at failure is determined for steel tubes with an outside diameter of 0.75 in. and a wall thickness of 0.035 in. in lengths ranging from 0.08 in. to 23.54 in. The steel had an ultimate tensile strength of about 140,000 psi. The tubes with length/diameter ratio (L/D) greater than 3.5 failed by plastic two-lobe buckling under a torque which was independent of the length. The tubes with L/D less than 1 failed by shearing at a torque which increased with decreasing length. Examination of Fuller's data obtained on steel tubes tested at Wright Field indicates that the effect of length on torque at failure is negligible, except for very short tubes, provided that the ratio of diameter/wall thickness ratio D/T is below a critical value which decreases from 80 for tubes with an ultimate tensile strength of 90,000 psi to 28 for tubes with an ultimate tensile strength of 200,000 psi. No data are given to indicate how these empirical relations might apply to other tubes than the steel tubes from which they were derived.

Walter Ramberg, USA

482. A. Boodberg and E. R. Parker, Transition temperatures of structural steels. Effects of geometry and welding on high-yield-strength steels, Weld. Res. Suppl. 14, 167–177 (Apr. 1949).

This paper describes tests on four high-yield-strength structural steels and one mild steel. Three types of specimens were used, a 3-in.-wide edge-notched specimen, a 12-in-wide centrally notched specimen, and a specimen that provided restraint to plastic flow at a corner produced by welding together steel plates set along three mutually perpendicular planes. Three different types of electrodes were used with different amounts of preheat to establish optimum welding conditions. The specimens were tested at various temperatures from +100 F to −150 F in order to determine the temperature at which the fracture changed from a ductile-shear type to the brittle-cleavage type. Transition temperatures for the large welded specimens varied from −65 F to +75 F. Transition temperatures for the notched specimens varied from +15 F to −108 F. The transition temperatures of the notched specimens were always about 50 F lower than those of the larger restrained specimens. This was true for all the steels, including the mild steel. In the welded restrained specimens fracture invariably started at nominal stress values that were below the ordinary yield strength of the material.

Evan A. Davis, USA

483. Pierre Laurent, Effect of the shape and the dimensions of the test bar on the fatigue limit (in French), Rev. Metall. 46, 55–59 (Jan. 1949).

According to the theory of Orowan, overstressed regions in a heterogeneous stress field progressively strain-harden and suffer less and less plastic flow during fatigue, reaching some limiting stress value less than or equal to the fracture strength of the crystal. The present paper attempts to justify this mechanism through the gradual accumulation of "catches" or "hooks" occurring at the ends of slip planes during stress reversal. These catches gradually act to limit slip and put the crystal under a state of hydrostatic tension, the magnitude of which is to be compared with the crystal fracture strength. The author feels that such a mechanism may serve as a guide to further investigations regarding specimen size and shape.

L. F. Coffin, Jr., USA

Design Factors, Meaning of Material Tests

(See Rev. 459)

Material Test Techniques

(See also Revs. 468, 577)

484. Noah A. Kahn and Emil A. Imbembo, Notch sensitivity of steel evaluated by tear test, Weld. Res. Suppl. 14, 153–166 (Apr. 1949).

The so-called Navy tear test, the application of which in testing various types of ship-plate steel is described in this paper, is another of the numerous comparative tests devised with the purpose of evaluating the notch-sensitivity of steel ship plates. It differs from the other tests in the shape of the specimen and the notch, as well as in the strain velocity produced by the eccentrically applied force. The authors attempt to prove by their test results that the relative evaluation, in terms of the transition temperature, of the different steels tested is satisfactory, and that therefore the test would constitute an effective procedure of comparative quality control of ship-plate material. No attempt is made to analyze the testing conditions in basic terms.

A. M. Freudenthal, USA

485. E. J. Ripling and G. Tuer, Apparatus for tensile testing at sub-zero temperatures, Prod. Engng. 20, 103–105 (Jan. 1949).

Details of carrying out tensile tests at subzero temperatures are described. Included is a description of tension grips claimed to produce a maximum eccentricity of 0.002 in. in loading. Here specimens with button-head ends are used in conjunction with split grips. Details of cooling are illustrated, by forcing liquid

nitrogen into an insulated chamber under pressure. A special knife-edge lever-type gage for measuring instantaneous diameters during the test is described. L. F. Coffin, Jr., USA

486. F. H. Müller, Instrumentation for the determination of stress-strain diagrams of single fibers under extreme conditions (in German), Kolloid Z. 112, 84–91 (Feb. 1949).

The author describes the construction and operation of a simple testing machine designed to indicate, with a fair degree of accuracy, the tensile properties of fine textile fibers. Fibers up to 20 mm in length may be tested. The dynamometer is an elastic system composed of two small-diameter steel rods; the deflection is measured by means of a telescope-mirror-scale optical system. This load-indicating system is essentially that described by R. L. Steinberger [Physics 5, p. 53 (1934)]. The loading may be applied at any desired rate by means of a motor-driven threaded spindle along which moves one of the fiber attachments. The extension of the fiber under test is determined from the known deflection of the load-indicating device and the motion of the fiber attachment along the threaded spindle. The extension of the fiber can be determined to 0.01 mm. Provision is made for immersing the test fiber in a liquid bath at any desired temperature. Numerous stress-strain graphs are included in the paper, illustrating the results obtainable with the test machine. The variation in the mechanical properties with temperature for similar fibers and the variation in the properties from fiber to fiber are vividly illustrated. George H. Lee, USA

Mechanical Properties of Specific Materials

(See also Revs. 451, 460, 467, 480, 484, 486)

487. P. O. Pashkov, Periodicity of deformation of plastic elongation and compression of coarse-grained steel (in Russian), Zh. tekh. Fiz. 19, 391–398 (Mar. 1949).

The material used was open-hearth steel with 0.45% C, after heating it to 1050 C, cooling to 680, keeping there for 1 hr and cooling in the air. It was coarse grained and consisted of pearlite with small boundaries of ferrite. For some experiments steel with 0.07% C was used.

A tensile-test specimen was halved along the axis, the planes were polished and etched and the single grains measured. After putting both parts together the specimen was broken, and the same grains were measured again. The elongations were scattered from 3% to 27% in the zone of uniform elongation (mean value 13%) and from 4% to 37% near the rupture (mean value 17%).

In other specimens the ratio of the dimension of the grains along the axis a to that perpendicular to the axis b was measured before and after the rupture. Before rupture it was found to be $0.5<a/b<2.0$; after the deformation in the zone of uniform elongation $0.75<a/b<2.25$, and near the rupture $1.25<a/b<6.0$.

The amount of the deformation changes along lines 45 deg to the axis from very low to higher figures. After compression, there were parts with a deformation of opposite sign to the mean deformation. The scattering of deformations increases with the amount of the total deformation.

The conclusion is that it is not possible to derive an exact theory of the deformation of a polycrystalline body on the basis of the deformation of a single crystal.

George Masing, Germany

488. P. W. Bridgman, Linear compressions to 30,000 kg/cm² including relatively incompressible substances, Proc. Amer. Acad. Arts Sci. 77, 189–234 (Apr. 1949).

This work represents another of the very valuable contributions to understanding of the compressibility of materials which Bridgman has been publishing for some time. The scope of the work is particularly significant covering 25 elements including such unusual ones as tantalum, columbium, germanium, thorium, and tellurium. He has also included numerous alloys and intermetallic compounds. He also gives data on various minerals having cubic lattices and several noncubic minerals. Three organic crystals, namely sucrose, rhamnose and gulonic lactose have also been included. The most striking data are those for nickel which shows a broken line for the pressure-strain curve, the shape of the curve for increasing pressure being practically identical with that for decreasing pressure. R. G. Sturm, USA

489. S. I. Ratner, Change of mechanical properties of metals under hydrostatic pressure (in Russian), Zh. tekh. Fiz., 19, 408–411 (Mar. 1949).

Tensile stress-strain curves have been recorded for copper, magnesium, beryllium-copper, and for several magnesium and aluminium alloys, both at atmospheric pressure and with superposed hydrostatic pressures up to 2200 kg/cm². Only in the case of copper was the stress-strain curve practically independent of the hydrostatic pressure; for the other materials, the yield stress was between 20 and 40% higher at a superposed hydrostatic pressure of 2200 kg/cm² than in the ordinary test. In the graphs reproduced, the stress-strain curves with hydrostatic pressure are very similar to the ordinary curves displaced vertically (to higher stress values). E. Orowan, England

490. Carl Benedicks, The influence of liquids on the strength and the strain of a solid rod (in Swedish) Tekn. Tidsk. 79, 97–100 (Feb. 12, 1949).

The effect of immersion on the tensile and bending strength of solids is discussed and related to the cohesive forces in the surface of the wetted body and the surface tension of the wetting liquid. It is theoretically deduced that the strength of the solid will vary inversely with the surface tension of the liquid. The results of some very interesting experiments are described and seem to confirm the theory well. According to the theory, the wetting of a porous solid shell reduces the ultimate strength more than the wetting of a homogenous solid, in accordance with experiment.

Ragnar Nilson, Sweden

491. Vincent T. Malcolm, Surface hardened stainless steels, Prod. Engng. 20, 84–87 (Jan. 1949).

Mechanical properties of stainless steels of either the austenitic or ferritic type, surface hardened by a process known as "Malcomizing," are reported. In this process parts are subjected to ionized ammonia gas at temperatures of from 920–1050 F, forming a case depth up to 0.010 in. of chromium nitrides. The nitrided surface assumes a state of high residual compression, and so enhances fatigue resistance while reducing pitting and tension cracks. Brinell hardness numbers up to 1000 may be reached, and elevated temperatures do not have too drastic an effect on the hardness. The process imparts considerable wear and erosion resistance without altering appreciably the chemical stability or physical properties of the stainless steels so treated.

L. F. Coffin, Jr., USA

492. R. L. Bickerdike and D. A. Sutcliffe, The tensile strength of titanium at various temperatures, Metallurgia 39 303–304 (Apr. 1949).

Fairly pure (99.68%) titanium samples were obtained by the magnesium-reduction process. After describing the procedures followed, the authors give the results of tensile strength and hard-

ness tests for room temperature, 300 C and 500 C. They think that slight impurities are responsible for an increase of more than double in the room-temperature hardness and a considerable increase in the tensile strength, in comparison with data obtained by previous investigators. A. J. Durelli, USA

493. **W. O. Sweeny, Haynes alloys for high-temperature service,** Trans. Amer. Soc. mech. Engrs. 69, no. 6, 569–581 (Aug. 1947).

The author tested various alloys at room and high temperatures, and compares the results obtained for thermal coefficients of expansion, thermal conductivities, age hardenings, endurance limits, ultimate strengths, yield limits, ductilities, modules of elasticity and creep rates. Several of the tests were run on cast and forged forms, and the author derives criteria which should prove useful to the designer of jet engines. A. J. Durelli, USA

494. **Georges Welter, Dynamic torsion of metals and alloys used in aircraft construction,** Metallurgia 39: Feb., 188–190; Mar., 253–256; Apr., 313–315 (1949).

This paper reports tests on elastic and plastic properties in torsion of 5 alloys: aluminum alloy 24S-T, Mg alloy AM57S, mild steel SAE 1020, Ni-Cr steel annealed, and Monel metal K as drawn. Solid cylindric specimens of $^5/_{16}$-in. diam were used. Relative angles of rotation were measured with an optical gage which allowed a minimum strain change of 4.4×10^{-6} in./in. to be measured. The investigations were chiefly concerned with small permanent deformations in the range 0–0.5%.

Dynamic loading was produced by weights falling on a lever attached to one end of the specimen, which was fixed in bearings at both ends. Basic data obtained were loading energies (computed from hammer weight and height) and permanent deformations. Conclusions reported concern: (1) effects of impact speed on microplastic deformations; and (2) effects of prior permanent deformation on plastic behavior under loading in opposite sense (Bauschinger effect). Comparative evaluations of the 5 alloys tested are given. For example, the aluminum alloy 24S-T exhibited much less Bauschinger effect than any of the others, with mild steel SAE 1020 exhibiting the most.

P. S. Symonds, USA

495. **T. N. Armstrong, Impact tests of pressure vessels at −320 F,** Weld. Res. Suppl. 14, 34–40 (Jan. 1949).

In order to determine how welded pressure vessels of 8.5%-nickel steel would behave under shock loading at very low temperature, several small vessels were constructed and subjected to impact when filled with liquid nitrogen. A carbon-steel vessel and a stainless-steel vessel also were included in the test. Each vessel was subjected to impact loading by dropping a weight from a height of 5 ft. Charpy impact tests were made on specimens removed from the walls of the vessels after tests. Results indicate that small vessels of either 8.5%-nickel steel or 304 stainless steel are capable of withstanding considerable shock at liquid-nitrogen temperature. The carbon-steel vessel shattered badly at the first blow. The fracture surface was crystalline with no indication of shear. H. M. Schnadt, Luxemburg

496. **H. Chossat, Relation between grain size of recrystallization of aluminum and its mechanical properties** (in French), C. R. Acad. Sci. Paris 228, 1344–1345 (Apr. 20, 1949).

Tests of recrystallization of aluminum during annealing are discussed, and grain size is plotted (for anneals of the same 30-min duration) as it varies with the temperature at which the anneal is made. This is shown for aluminum 99.99% pure and for aluminum with small amounts of iron, magnesium, silicon, and zinc.

Next there is given a plot of elastic limit, ultimate strength, and elongation for aluminum 99.99% pure as functions of grain size. W. C. Johnson, Jr., USA

497. **R. Jacquesson and P. Laurent, Information given by fatigue tests on the crystalline state of sheet metal** (in French), Rev. Metall. 46, 89–101 (Feb. 1949).

Torsional fatigue tests were made on sheet specimens of commercially pure (99.5%) aluminum: (a) annealed, and (b) rolled half hard. In the half-hard state, maximum lifetime was found for specimens cut parallel to the direction of rolling, and much shorter lifetime for specimens cut at angles 45–90 deg from this direction.

Changes in appearance of the metal surface, as the fatigue test progressed, were observed microscopically (the flat surfaces having been electrolytically polished). Detailed descriptions are given of surface appearance and some 27 photomicrographs shown in illustration. Both for annealed and for rolled metal, lines were observed at early stages in each test. In test pieces of annealed aluminum, these "slip lines" were correlated with orientation directions of individual grains. However, for specimens of rolled sheet, the lines of discontinuity were generally parallel to directions of shear stresses whatever the orientation of a grain showing them. The annealed specimens showed a decohesion between grains preceding fracture; this type of breaking up was not observed in the hard-rolled sheet specimens.

Horace Grover, USA

498. **Ya. M. Potak and V. V. Sachkov, On the effect of ferrite-grain dimensions upon the strength of iron and steel at brittle failure** (in Russian), Zh. tekh. Fiz. 19, 399–407 (Mar. 1949).

The resistance of iron to brittle fracture depends very little on the temperature while the resistance to plastic shear fracture increases strongly with decreasing temperature. At low temperatures it is possible to avoid plastic deformation completely.

Specimens of armco iron with different grain size, as produced by a critical deformation and annealing, have been broken at −196 C in the tensile test. The influence of the linear dimensions m in microns upon the tensile strength σ can be represented by the formula

$$\sigma = 1360m^{-1/2}(1 - 0.225\sqrt{\lg V.10^9/m^3} - 0.55$$

where V is the volume of the specimen. It is derived on the basis of the statistical probability of notch flaws as starting the fracture on the assumption that the influence of a notch with a length proportional to m is proportional to $m^{1/2}$.

The influence of grain size upon the shear fracture as measured at room temperature is negligible within the limits of the experimental error.

A few experiments show that the brittle fracture starts from the crystals of ferrite also in the normalized steel, and in the incompletely quench-hardened steel. The broken ferrite crystals always show Newman bands. It is supposed that the fracture begins in these bands. George Masing, Germany

499. **G. A. Mellor and R. W. Ridley, The creep strength at 200 C of some magnesium alloys containing cerium,** J. Inst. Metals 16, 679–692 (Apr. 1949).

Alloys containing mixtures of rare elements have valuable properties at high temperatures. Measurements are reported for Mg alloys with addition of up to 6% cerium and "mischmetal." Creep strain and creep rate were measured at 120 hr

and 200 C with a stress of 3.15 kg/mm². Factors investigated were Ce content, mechanical and heat-treatment, and aging. Increasing the Ce content from 2 to 6% does not give much further improvement. B. Gross, Brazil

500. Giuseppe Rinaldi, Test on an expansive concrete beam (automatic prestressing) (in Italian), G. Gen. civ. 87, 385–389 (July-Aug. 1949).

Results of comparative tests on three beams of the same shape and span, made of prestressed concrete, expansive concrete (automatic prestressing), and plain reinforced concrete. Best results were obtained from the prestressed concrete beam. The load necessary to start cracking of the expansive concrete beam was however only slightly less than the load required by the prestressed concrete beam. The expansive concrete beam withstood the highest load before failure. A. J. Durelli, USA

501. Adrien Dubuisson, Evaluation of the hydraulic value produced by the addition of slag cement in the metallurgical cements manufacturing (in French), Rev. Matér. Constr. 1949: no. 403, 113–116; no. 404, 149–153.

The principal aim of the investigation is indicated in the continuation of the title, viz., "and evaluation of the probable strength of these cements from a determined specific surface," this first paper (to be continued) being mainly chemical. As a preliminary the strength of clinker-cements during different periods of hardening is related by "practical and mnemotechnic" formulas to the specific surface. An attempt is then made to judge the influence of the specific surface in metallurgical cements in accordance with the mean specific surface of the mixture, without taking account of the differences in fineness between clinker and slag. The strength at any period is related to the strength at one day and a limiting strength. M. Reiner, Israel

502. E. W. Scripture, Jr., and F. J. Litwinowicz, Some factors affecting air entrainment, J. Amer. Concr. Inst. 20, 433–442 (Feb. 1949).

This is an investigation into the influence of (1) slump, (2) cement factor, and (3) sand to total-aggregate ratio on the amount of air entrained in concrete mixes. In general, entrained-air content decreases with increasing (2), increases with increasing (3), but is only slightly affected by (1). M. Reiner, Israel

503. M. D. Burdick, R. E. Moreland, and R. F. Geller, Strength and creep characteristics of ceramic bodies at elevated temperatures, Nat. adv. Comm. Aero. tech. Note no. 1561, 53 pp. (Apr. 1949).

The creep tests described in this paper were made on threaded tensile specimens of about 0.3-in. diam and about 4-in. gage length. The grips were made of ceramic materials. Six different ceramic bodies, prepared as possible turbine-blade materials, were tested at temperatures ranging from 1500 F to 2050 F. Four different methods of loading were used, but most of the tests were so-called "step tests." In the step tests either the load or the temperature was increased by given amounts at intervals of about 200 hr. The creep curves and tabular data are given. Additional data on bulk density, linear thermal expansion, modulus of elasticity, and the weakening effect of water vapor are also given. Evan A. Davis, USA

504. Helmut G. F. Winkler, Clays and their properties—An interpretation, Research Lond. 2, 175–183 (Apr. 1949).

Clays represent a large group of very fine mineral aggregates which can generally be made plastic by the addition of water and which have been found, with a few exceptions, as sediment of the finest mud in water. Because of recent advances in techniques of mechanical, X-ray, and thermal analyses, a better understanding is now available of those common but complex materials. It is now known that the mineral constituents of clays can be arranged in four groups: those formed during weathering, those resisting weathering, biogenic constituents, and minerals formed after sedimentation. The properties of clays depend not only upon their mineral constituents, but also on the grain size. Such properties as bleaching characteristics, base-exchange capacity, plasticity, and thixotropy (a very interesting process whereby a clay suspension gels to a solid form when left stationary, but returns to the liquid form upon mechanical agitation) are discussed. S. S. Manson, USA

505. H. Korth, The use of fly ash as structural material (in German), Bauplan. Bautech. 3, 125–132 (Apr. 1949).

Brown-coal fly ashes when mixed with water harden in air due to a combination of hydraulic cementing action, gypsum-like binding, and lime-like carbonization. The hardenability and relative participation of the three described mechanisms depend upon chemical composition, fineness, and degree of fusion. The tentative German norm DIN4209 established two quality classes, Br 40 and Br 10, giving respective 28-day cube compression strengths of 40 and 10 kg per sq cm.

In 1947 the magistrate of Greater Berlin suggested use of this cement in permanent structures in (1) masonry mortars, (2) interior and exterior wall plasters, and (3) manufacture of building slabs (100 × 33 × 5 cm). The author critically discusses the properties of fly-ash mortars, plasters, and other compositions in comparison with those of the commonly employed standard materials for the same class of work; he concludes that the principal field of fly-ash utilization is in masonry mortars and wall plasters, and that special high-quality ashes, which can be utilized in the manufacture of structural units with higher strength and heat-insulation requirements, are available only in restricted quantities. Hans F. Winterkorn, USA

Mechanics of Forming and Cutting

(See also Rev. 591)

506. E. Siebel, The application to shaping processes of Hencky's laws of equilibrium, J. Iron and Steel Inst. 155, 526–534 (1947); (in German) Ingen.-Arch. 16, 164–172 (1948).

Using the theorems of Hencky and Prandtl concerning the geometric properties of the net of slip lines in problems of plane plastic strain, the author presents a qualitative discussion of important forming processes (forging, rolling, drawing and extruding). While the slip-line patterns used by the author satisfy the static boundary conditions, they violate, in most cases, equally important kinematic boundary conditions. W. Prager, USA

507. L. A. Carapella and W. E. Shaw, A process of augmenting cold-drawability of the magnesium +1.5 per cent manganese alloy, Trans. Amer. Inst. min. metall. Engrs. 171, 277–285 (1947).

The maximum cold-drawability of magnesium with 1.5% manganese alloy is about 25% when the conventional method of applying the load in a progressively increasing way is employed. The authors applied the load intermittently, increasing it after each step, and claim that with this method the drawability is increased up to 40%. A theoretical explanation of the phenomenon is attempted and a drawing of the deep-drawing unit used is given. A. J. Durelli, USA

Hydraulics; Cavitation; Transport

(See also Revs. 406, 419, 510, 518, 564)

508. Alb. Schlag, Conduit flow of liquids, gases and vapors. Laws of flow. Measurement of discharge (L'écoulement en conduites des liquides, gaz et vapeurs...), Paris, Dunod, 1949, 254 pp. Paper, 7.5 × 5 in., 81 figs., no price stated.

An extensive exposition of laws and formulas pertaining to conduit flow, without theoretical or mathematical derivations.

Ed.

509. John W. Forster and Raymond A. Skrinde, Control of the hydraulic jump by sills, Proc. Amer. Soc. civ. Engrs. 75, 469–483 (Apr. 1949).

For any supercritical discharge (shooting flow) in a rectangular channel, a hydraulic jump may be created and prevented from receding downstream by either a weir, extending across the channel, or an abrupt rise in the channel bottom. Both cases can be treated analytically by the equation of momentum, as shown by the author. Experimental research work has proved that the accuracy of this equation is well sufficient for any practical purpose, and that it is possible, for an expected range of discharge, to calculate the minimum height of sill necessary to prevent the jump from being washed out.

It is worth while to mention that this problem, or very similar ones, have been investigated by Charles Jaeger [Wasserkraft und Wasserwirtschaft 31, no. 24 (1936); Rev. Gén. Hydr. nos. 37–41 (1947); Rev. 1, **1283**], L. Escande [Rev. Gén. Hydr. nos. 25 and 26 (1939)] and I. Léviant [Rev. Gén. Hydr. 13, nos. 38 and 40 (1947); Rev. 1, **553**].

Charles Jaeger, England

Incompressible Flow: Laminar; Viscous

(See also Revs. 395, 396, 397, 508, 536, 550, 553, 556, 557, 565, 575, 587, 590)

510. N. E. Kochin, I. A. Kibel, and N. V. Roze, Theoretical hydrodynamics (Teoreticheskaya gidromekhanika), Vol. 1, Fourth edition, revised. Leningrad-Moscow, Ogiz-Gostekhizdat, 1948, 535 pp. Paperboard, 8.6 × 5.7 in., 179 figs., $2 (at Four-Continent Book Corp.).

This is an advanced and comprehensive textbook presenting the classical ideal incompressible-flow topics in a mathematical treatment. The titles of the chapters are: 1. Fluid kinematics, 35 pp. 2. Basic equations of ideal fluids, 37 pp. 3. Hydrostatics, 27 pp. 4. The simplest flows, 34 pp. 5. Vortical flows, 93 pp. 6. Plane flows, 117 pp. 7. Spatial flows, 42 pp. 8. Wave motion of ideal fluids, 133 pp.

In chapter 5 the von Kármán vortex street is treated in much detail. In Chapter 6 flow separation, thin airfoils, and the planing plate are given a particularly detailed mathematical treatment (the methods of Kirchhoff, Joukovsky-Mitchell, and Levi-Civita are presented). Chapter 8 is quite comprehensive and includes also a discussion of waves in a rotating atmosphere.

Ed.

511. R. Berker, On the kinetic energy of a viscous incompressible fluid in a limited spatial domain (in French), C. R. Acad. Sci. Paris 228, 1327–1329 (April 20, 1949).

It is shown that the kinetic energy E of a viscous incompressible fluid in a bounded domain is dissipated at least at an exponential rate in accordance with the relation $E \leq E_0 \exp(-16\nu t/D^2)$ where ν is the kinematic viscosity and D a diameter of the domain. The above inequality is derived by employing well-known expressions for the energy and the rate of dissipation of energy in a viscous incompressible fluid, and by applying Schwartz's inequality to the former of these expressions.

Louis Landweber, USA

512. S. S. Byushgens, The geometry of a stationary flow of an ideal incompressible fluid (in Russian), Izv. Akad. Nauk SSSR, Ser. Mat. no. 12, 481–521 (1948).

The flow under study is subjected to a conservative force (potential U). The method used is that of the mobile trihedron, in Cartan's ω-notation.

A number of conditions on these quantities and on the total energy $H = {}^1/_2V^2 + U + p/\rho$ are geometrically interpreted. Examples are the equations grad $H = 2V \times \omega$, $(V \text{ grad})\ \omega = (\omega \text{ grad})\ V$, the family of surfaces of constant total energy and the streamlines which are situated on them, also the case in which the magnitude of the velocity vector is constant. Other cases are that in which $H = f(z)$, hence the surfaces of total energy are parallel planes, and that of so-called minimal ($p_2 = q_1$) and rectilinear congruences of streamlines.

The paper ends with a discussion of the spiral flow, that is, a flow for which at every point the direction of the vortex vector coincides with the direction of the velocity ($\omega = kV$). The type of theorem which is derived in this paper can be gathered from the following example: If the congruence of streamlines consists of straight lines, then it is either a minimal congruence or it is normal to a family of parallel surfaces, and in this second case the vortex lines are orthogonal to the streamlines.

Courtesy of Mathematical Reviews D. J. Struik, USA

513. R. A. Tyler, The available theoretical analyses of two-dimensional cascade flow, Nat. Res. Counc. Canad. Aero. Note no. AN-4, 16 pp. (1949).

Certain methods are described for the calculation of the steady, two-dimensional, potential flow of an incompressible, inviscid fluid past cascades of airfoils. Quite complete analytical detail is given in most cases. A unified approach is thus applied to many existing calculation methods.

The calculation of the flow past a given airfoil cascade is illustrated by methods due to Howell, Garrick, Mutterperl and Katzoff, Finn and Laurence. The calculation of the flow and airfoil shape corresponding to a chosen airfoil-surface pressure distribution is described by a method due to Goldstein and Jerison. A discussion is given of approximate methods for thin airfoils (Diesendruck) and for closely spaced airfoils (Ackeret). Methods leading to special classes of airfoils (Merchant and Collar, and Lighthill) are discussed. A compressible theory (Spurr and Allen) of thin airfoils, widely spaced and unstaggered, is considered. An extensive bibliography is included.

The important early work of F. Weinig (*Die Strömung um die Schaufeln von Turbomaschinen*, Leipzig, Barth, 1935) is largely neglected. No mention is made of Weinig's methods for the arbitrary pressure distribution problem, for various special classes of airfoils, and for cambered plates by approximate means.

W. G. Cornell, USA

514. Max Shiffman, On free boundaries of an ideal fluid. II, Commun. appl. Math. 2, 1–11 (Mar. 1949).

This article, continuing with the method developed in the first article of the same title [same source, 1, 89–99 (1948); see Rev. 1, **861**], takes up the force on an obstacle in a flowing incompressible fluid due to formation of a cavity behind the obstacle. The principle of reflection across free boundaries is applied to several examples both symmetric and unsymmetric. It is emphasized that the geometric interpretation of this force and of the drag

coefficient makes the drag coefficient easily calculable. The case of a cavity in which a backward spout is formed is considered and solved both for the symmetric and for the unsymmetric case. The methods used are demonstrated to be generally applicable.

R. G. Wilson, USA

515. D. N. de G. Allen, The formation of closed wakes in fluid motions, Quart. J. Mech. appl. Math. 2, 64–71 (Mar. 1949).

This paper is concerned with the analytical determination of two-dimensional, potential flow patterns containing a closed region bounded by free streamlines. Examples of such patterns, as distinguished from the classical solutions of Helmholtz, Kirchhoff, and Rayleigh for flows with free-streamline regions open to infinity, have been obtained previously by Southwell and Vaisey using relaxation techniques [Phil. Trans. Roy. Soc. A, 240, p. 117 (1946)]. The author demonstrates here that it is also possible to obtain patterns of the desired general type analytically by extension of the methods of conformal transformation employed in the classical studies. As an example, a treatment is given of the flow in a right-angle corner when there is a dead-air region of arbitrary constant pressure in the corner.

Walter Vincenti, USA

516. L. N. Sretenskiĭ, On annular waves on the surface of a rotating liquid (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk 1949, no. 1, 5–18.

Periodic annular waves upon the surface of a fluid rotating in a circular cylinder of infinite depth are considered for two cases. In the first case the fluid is rotating as a rigid body, giving a paraboloid as equilibrium surface. The treatment is restricted to small waves and small angular velocities. It is found that the effect of the rotation is to increase the frequencies over those for a nonrotating fluid. The second case considered is one in which the axis is a vortex with circulation Γ (velocity of a particle at distance r is $\Gamma/2\pi r$). To avoid the singularity on the axis the fluid is enclosed between two concentric cylinders. It is found again that the frequency is increased over that for a nonrotating fluid but that the amount of increase decreases as the ratio of the radii of the outer to the inner cylinder increases.

J. V. Wehausen, USA

517. V. V. Sokolovskiĭ, On the equations of nonlinear filtration (in Russian), Doklady Akad. Nauk SSSR 65, no. 5, 617–620 (Apr. 1949).

After the explanation of the usual equations of seepage the author chooses the following nonlinear relation between the velocity v and the hydraulic gradient i: $ki = v/[1 - (v/m)^2]$ where k and m are constants. This equation differs from Darcy's law in such a way as to allow for the effects of turbulent flow.

The differential equations of seepage are then subjected to several transformations involving complex variables. The results are left in complex form, and it is claimed, without demonstration, that they make possible the actual solution of various seepage problems.

Alexander Hrennikoff, Canada

518. J. W. Daily, Cavitation characteristics and infinite-aspect-ratio characteristics of hydrofoil section, Trans. Amer. Soc. mech. Engrs. 71, 269–284 (Apr. 1949).

This report is concerned with a two-dimensional investigation of an NACA 4412 airfoil in a water tunnel. The growth of cavitation as influenced by velocity, submergence, and angle of attack is studied. The relation between the angle of attack and the value of the cavitation parameter at which the inception of cavitation occurs, is shown for each face of the hydrofoil and is compared with calculated values. Convenient curves are given showing submergence required to avoid cavitation for different velocities and angles of attack. Measured hydrodynamic characteristics in the absence of cavitation are compared with NACA data on wind-tunnel tests of finite aspect-ratio span.

H. Julian Allen, USA

519. M. M. Nicolson, The interaction between floating particles, Proc. Camb. phil. Soc. 45, 288–295 (Apr. 1949).

The shape of the free surface of a liquid in the neighborhood of a floating bubble, and the interaction between two equal bubbles are analytically determined by the hypothesis that the submerged portion of the bubbles remains spherical (rather little bubbles). The latter results are plotted in potential-energy-distance curves and compared with the curves of the interaction between inert gas atoms.

Duilio Citrini, Italy

Compressible Flow, Gas Dynamics

(See also Revs. 561, 584)

520. R. K. Tempest, Some physical interpretations of potentials representing supersonic motion of compressible fluids, Proc. Camb. phil. Soc. 45, 246–250 (Apr. 1949).

This note attempts to explain the meaning of "supersonic sources and doublets" from a physical point of view. Potentials of doublets are suggested to be subdivided into primary and secondary potentials. The potential of a doublet as suggested by Prandtl is called a secondary doublet. It is shown that the effect of two equal and opposite line distributions of point sources, or a line distribution of primary doublets, is exactly the effect of equal and opposite secondary doublets of the same strength placed at the ends of the distribution.

C. T. Wang, USA

521. R. C. Prim, On the uniqueness of flows with given streamlines, J. Math. Phys. 28, 50–53 (Apr. 1949).

A given flow is called unique when the only flows with the same streamline pattern have velocity fields simply proportional to that of the given flow.

Through the analysis of the equations governing steady flow of ideal fluids in the absence of body forces, it is established that any such flow is unique unless it has a constant velocity magnitude along each individual streamline. The geometric implications of this result in the cases of irrotational flows, of plane flows and axially-symmetric flows are clarified.

Giulio De Marchi, Italy

522. W. Richter, One-dimensional stationary flow at constant pressure in moving systems (in German), Ingen.-Arch. 16, 422–445 (1948).

In connection with studies on "constant pressure" compressors (e.g., axial compressors), the author investigates the one-dimensional compressible flow along a curve in a plane, a cylindrical surface or a surface of revolution with body force but with constant pressure. The plane is assumed either to move with a constant velocity in its plane or to rotate around an axis perpendicular to the plane. The cylindrical surface is assumed to move with a velocity parallel to its generator. The surface of revolution is assumed to move along its axis or rotate around its axis. The square of the absolute velocity of the fluid is assumed to vary essentially linearly with the distance from the origin for relative plane or a cylindrical surface, from the axis for relative motion in a surface of revolution. The problem is to calculate the fluid path (which is the "blade shape" in a compressor). It turns out that,

by using simple transformations, all problems stated above can be reduced to the first problem: motion in a plane which is itself in linear motion. Analytical and graphical solutions are given.

H. S. Tsien, USA

523. A. J. Eggers, Jr., One-dimensional flows of an imperfect diatomic gas, Nat. adv. Comm. Aero. tech. Note no. 1861, 32 pp. (Apr. 1949).

The author examines the deviation of the one-dimensional flow of a real gas from that of a perfect gas by adopting Berthelot's equation of state

$$p = \rho RT/1(1 - b\rho) - c\rho^2/T$$

where p is the pressure, ρ is the density, T the absolute temperature, R the usual gas constant, and b and c are two additional constants for molecular size and intermolecular forces respectively. It is further assumed that the specific heat is

$$c_v = c_{v_i}\left\{1 + (\gamma_i - 1)\left(\frac{\theta}{T}\right)^2 \frac{e^{\theta/T}}{[1 - e^{\theta/T}]^2}\right\} + \frac{2c\rho}{T^2}$$

where c_{v_i} and γ_i are values for an ideal gas, and θ is a characteristic temperature for the specific heat of oscillators. During the motion, the gas is assumed to exhibit no heat-capacity lag, shock-free flows are considered isentropic, and flow through plane shock adiabatic.

The special cases of flow through normal and oblique shocks in free air at sea level are investigated. It is found that up to Mach numbers of 10 the pressure ratio across a normal shock differs little from its ideal gas value; whereas at Mach numbers above 4 the temperature rise is considerably below, and hence the density rise is well above that predicted assuming ideal gas behavior. The effects of gaseous imperfections on oblique shock flows are studied from the standpoint of their influence on the lift and pressure drag of a flat plate operating at Mach numbers of 10 and 20. The influence is found to be small.

C. C. Lin, USA

524. R. Timman, Asymptotic formulae for special solutions of the hodograph equation in compressible flow (in English), Nat. LuchtLab. Amsterdam Rap. no. F. 46, 26 pp. (Apr. 22, 1949).

The author presents a general mathematical treatment for obtaining approximate solutions of hodograph equation in compressible flow. The method consists in transforming the hodograph equation into a form for which approximation solutions may be written by analogy with a somewhat simpler differential equation. The method is illustrated first by application restricted to subsonic flow where a solution is obtained in terms of asymptotically convergent series. Then the more general transonic case is treated and approximation formulas are derived which are valid in the subsonic range and in the supersonic range below the maximum velocity.

The main steps in this procedure appear to be:

1. Write down the hodograph equation for the velocity potential using the well-known Legendre transformation. This provides a linear differential equation in terms of the velocity, q, and the velocity direction, θ, as variables.

2. This differential equation is then transformed to correspond to the general form (1) $v'' + [P(z) + \theta(z)]\, v = 0$.

3. Various methods of obtaining approximate solutions of equation (1) are available, in general leading to hypergeometric series, one of the methods having been developed by Langer. The author generalizes the Langer treatment for application to the hodograph equation of compressible flow. Several theorems are proved so that approximate solutions to equation (1) may be written out by using known solutions to the simpler differential equation (2) $u'' + P(z) = 0$.

As is to be expected, the method is rather elaborate since it requires first the derivation of the functions $P(z)$ and $\theta(z)$ symbolically included in equation (1) from the actual functional variables appearing in the hodograph equation, and second, the procedure for making these functions satisfy the boundary conditions of the problem involved. In the transonic case one then obtains a series expression defining the solution which may be obtained. The solutions, in turn, involve Bessel functions and for the evaluation of the potential these functions are replaced by asymptotic expressions. The results obtained for several limiting cases (i.e. for q small and for large values of the index of the hypergeometric series involved) and compared with results obtained elsewhere by Garrick and Kaplan, and by Cherry and Lighthill.

F. K. Hill, USA

525. F. Reutter, On an approximate quasi-linear potential equation of plane compressible flow and its solutions obtained by Legendre transformation (in German), Z. angew. Math. Mech. 25/27, 156–157 (1947).

The author gives the differential equation for the velocity potential of two-dimensional transonic flows first derived by von Kármán [J. Math. Phys. 26, 182–190 (1947); see Rev. 1, **508**]. This equation is linearized by Legendre transformation into variables U, V, the components of the disturbance velocity. The undisturbed flow is in the direction of the x-axis. The resultant equation is that of F. Tricomi [Atti Accad. Lincei Mem. Cl. Sci. Fis. Mat. Nat. (5) 14, 133–247 (1923)]. The solution of this equation can be written as an infinite series each term of which is the product of a hyperbolic cosine in V and a function in U, in fact, as pointed out by von Kármán, Bessel functions of order $^1/_3$. For flows over a body, the solution must have a singularity at the point corresponding to the conditions at infinity. The author suggests a scheme of approximate solution for a given profile without horizontal slope, symmetric with respect to the x-axis, by satisfying the boundary conditions at only a finite number of points.

H. S. Tsien, USA

526. M. Schaefer, The appearance of a compression shock in the neighborhood of a convex wall surface which is free from singularities, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1206-1A, 22 pp. (1949).

The German original of this appeared in 1944. In it, the author presents what is stated to be the first calculated example of a flow over a convex wall in which a shock wave is present despite the complete absence of surface singularities in the neighborhood of the wave. To assure the presence of a shock wave, the calculation presupposes an envelope of Mach lines beginning at a point in the interior of the flow field and then determines a convex boundary consistent with the existence of this envelope. The calculation in the vicinity of the initial point of the envelope is carried out analytically on the basis of previous work by the same author (same source, no. F-TS-1203-1A, 1949). A streamline some distance from the initial point in the analytical region is then chosen as a boundary and the flow field extended upstream to the sonic line by judicious application of the standard method of characteristics. By this means (and after three trials), a finite supersonic field is determined which resembles in many respects the type of supersonic region known to occur in transonic flows over convex boundaries. The question of whether or not this region can be embedded in a physically possible subsonic field, however, is not discussed.

Because of omissions in the original paper from which the translation was made, the figures giving the important final re-

sults are missing from the present text. They can be found, however, by reference to the article by W. Tollmien in Section C 4.2 of the AVA Monographs (British Min. of Aircraft Prod., Repts. and Trans. no. 997, 1948). Walter Vincenti, USA

527. F. I. Frankl', Asymptotic resolution of Chaplygin's functions, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1212-1A, 7 pp. (1949) = Doklady Akad. Nauk SSSR 58, 757–760 (1947).

The stream functions of an irrotational compressible flow in hodograph variables q and θ can be written as $\psi_n = Z_n(\tau)\sin 2n\theta$ where $\tau = q^2/q^2_{max}$. For transonic flows, it is important to know the behavior of $Z_n(\tau)$ for τ near τ^*, which is the "critical" value for τ corresponding to unit local Mach number. This note proves the following asymptotic formula for the subsonic range $0 \leqslant \tau \leqslant \tau^*$

$$\frac{Z_{n/2}(\tau)}{Z_{n/2}(\tau^*)} = \lambda(n^{2/3}\eta) + \frac{\lambda'(n^{2/3}\eta)}{n^{2/3}} + \ldots + \frac{\lambda^{(k)}(n^{2/3}\eta)}{n^{2k/3}} + \frac{\Lambda_n^{(k+1)}(n^{2/3}\eta)}{n^{(2k+1)/3}}$$

where

$$\eta^{3/2} = \tfrac{3}{4}\,{}_{\tau}\!\int^{\tau^*}(1-\tau/\tau^*)^{1/2}(1-\tau)^{-1/2}\,d\tau/\tau.$$

The function $\lambda(\xi)$ is Airy's function defined by the conditions

$$\lambda''(\xi) - \xi\lambda(\xi) = 0, \quad \lambda(0) = 1, \quad \lambda(+\infty) = 0$$

The functions $\lambda^{(i)}(\xi)$ are defined by the conditions

$$\lambda^{(i)''}(\xi) - \xi\lambda^{(i)}(\xi) = q_i(\xi),$$

$$\lambda^{(i)}(0) = \lambda^{(i)}(\infty) = 0,$$

were $q_i(\xi)$ is a known function. The remainder term satisfies the following relations:

$$|\Lambda_n^{(k+1)}(\xi)| < A^{(k+1)}, \quad |\Lambda_n^{(k+1)\prime}(\xi)| < A^{(k+1)}.$$

H. S. Tsien, USA

528. Mark Lotkin, Supersonic flow over bodies of revolution, Quart. appl. Math. 7, 65–74 (Apr. 1949).

Equations are derived by which the steady symmetric flow near the nose of a pointed projectile might be computed as a perturbation of the conical flow near the tip of a circular cone. The characteristic curves of the perturbation equations are those of the original conical flow. The flow involves first-order vorticity, which must be considered, due to the perturbation of the nose shock wave. The computational procedure is outlined in some detail, but no example of its use is presented. It is difficult for the reviewer to assess the value of this theory in comparison with a straightforward computation by the method of characteristics. W. R. Sears, USA

529. G. N. Ward, Supersonic flow past slender pointed bodies, Quart. J. Mech. appl. Math. 2, 75–97 (Mar. 1949).

The flow over a pointed body of general cross section at supersonic velocities is considered under the assumptions of the usual perturbation theory. The particular solutions of the resulting linearized potential equation in the "Heaviside operational form" consist of products of modified Bessel functions of argument r and trigonometric functions of θ (r and θ are the radial and angular coordinates in a cylindrical coordinate system whose axial direction is in the free-stream direction). These particular solutions when specialized to the vicinity of the body, i.e., for small values of r, reduce to harmonic functions. The over-all drag and lateral force coefficients are then determined by using a suitable superposition of the above harmonic functions in the calculation of the flux of momentum through a cylindric control surface close to the body. The results are found to depend only upon two of the coefficients of the superposition; these coefficients are then found from the given distribution of the cross-sectional area by considering the boundary condition at the surface of the body.

The main application of the above results concerns the flow over a body of revolution with wings of small aspect ratio. (Here the wings are considered to be a limiting form of a slender body with an elliptical cross section.) For two special configurations it is possible to obtain simple expressions for the wing-body interference. Hideo Yoshihara, USA

530. J. B. Broderick, Supersonic flow round pointed bodies of revolution, Quart. J. Mech. appl. Math. 2, 98–120 (Mar. 1949).

A method is presented for solving the complete equation for isentropic axisymmetric flow about bodies of revolution to a higher order of approximation than has previously been obtained. The potential is expanded in powers of t and $\log t$ (t thickness ratio) and the coefficients in this series are then expanded in powers of r and $\log r$, with sufficient terms being obtained to give the potential correct to the order t^4 on the body. The results give the pressure and drag coefficients of the body correct to the order t^4. As an illustration the method is applied to cones of semiangle 5 deg, 10 deg, and 15 deg. For the 5-deg cone the results are in almost perfect agreement with exact numerical solutions up to a Mach number of 5. Good agreement for the 10-deg cone was obtained up to a Mach number of about 3.0. For the 15-deg cone the agreement was good only at Mach numbers below about 1.4.

The mathematical work involved in applying this method—even for bodies of simple shape—is extensive and tedious. Aerodynamicists will probably prefer to use the method of characteristics for the calculation of precise theoretical pressure distributions. John V. Becker, USA

531. J. B. Broderick, Supersonic flow past a semi-infinite cone, Quart. J. Mech. appl. Math. 2, 121–128 (Mar. 1949).

The present paper may be considered an extension of the preceding one. Its main object is to consider the influence of a shock wave, and show that the present solution gives the same approximate results as the earlier theory. The extended theory has been restricted to the semi-infinite cone because of the added complications.

The procedure is to develop two series representations, one about the axis of the cone (similar to the earlier theory), and the other about the shock wave; the two solutions are then matched along an appropriate line. The solution about the shock wave is represented by a series in terms of $\beta_1 - \alpha_1$, where α_1 and β_1 are tangents of the Mach angle and the angle between the shock wave and the axis of the cone, respectively.

The essential result is that the present theory (taking into account the shock) agreed with the earlier theory (isentropic case) to the order t^4, t being the semiangle of the cone.

Hideo Yoshihara, USA

532. M. J. Lighthill, Supersonic flow past slender bodies of revolution the slope of whose meridian section is discontinuous, Quart. J. Mech. appl. Math. 1, 90–102 (1948).

The linearized theory of slender bodies of revolution in supersonic flow, either symmetric or yawed, is ordinarily based on the assumption that the meridian section is a continuous curve with continuous slopes [von Kármán and Moore, Trans. ASME 54, 303–310 (1932)]. This is an attempt to extend the theory to shapes having discontinuous slopes. Instead of

$$f(x) = -S'(x)/2\pi,$$

where $S(x)$ is $\pi R^2(x)$, the cross-sectional area distribution, the author uses for the source strength distribution $f(x)$ the function

$$f(x) = -(2\pi)^{-1} {}_{a_0}\int^x S''(y)dy - \Sigma f_i(x),$$

where $x = a_0$ denotes the nose, and the integral is a Lebesgue integral which ignores the points $y = a_i$ where the integrand is not defined. Each $f_i(x)$ is zero for $x < c_i$ and continuous for $x > c_i$, where $c_i = a_i - \alpha R(a_i)$, α being the cotangent of the Mach angle. The author then deduces that there are extra terms, arising from the discontinuities, in the expressions for surface pressure and drag, that each discontinuity affects markedly the pressures in the region just behind it, and that these new contributions to the drag coefficient decrease as the Mach number increases. (If there are no discontinuities, the drag coefficient is independent of Mach number.) As an example, a double-cone projectile is considered, and numerical results are given. A similar investigation is made for the yawed projectile. Although the author again finds contributions to the pressure distribution due to the discontinuities, they integrate out in the lift and moment.

W. R. Sears, USA

533. Eric Reissner, On compressibility corrections for subsonic flow over bodies of revolution, Nat. adv. Comm. Aero. tech. Note no. 1815, 9 pp. (1949).

This paper is concerned with the form of the compressibility corrections for subsonic flow which follow from the linear perturbation theory. Although the essential differences between the compressibility corrections for two-dimensional flow and the corresponding corrections for axisymmetric flow about slender bodies of revolution are now known, the relation between the two cases is clearly demonstrated here by a study of the flow past an infinitely long corrugated cylinder. The solution obtained contains as limiting cases both the Prandtl-Glauert correction for two-dimensional flow and the Göthert correction for axisymmetric flow. In addition, it shows the nature of the transition from one limiting case to the other.

Included in the paper are velocity-correction formulas for a cylinder with a single bump and for a corrugated cylinder in the presence of walls.

J. S. Isenberg, USA

534. W. E. Moeckel, Use of characteristic surfaces for unsymmetrical supersonic flow problems, Nat. adv. Comm. Aero. tech. Note no. 1849, 49 pp. (Mar. 1949).

A numerical method is proposed for computing supersonic flow fields around unsymmetric bodies. For pointed bodies, the velocity field must be given near the vertex to provide a starting point for the computations. Shock waves are replaced by isentropic compressions to permit use of the equations of compressible potential flow. The velocity components are calculated at a network of points (defined by intersections of characteristic surfaces with each other) throughout the flow field using the difference equations obtained from the differential equations for the velocity potential. In a typical computation, three known neighboring points in the network are used to compute the flow at a fourth point. This method can be applied equally well to the nonlinear or linearized differential equation.

This reviewer believes that the families of characteristic surfaces chosen by the author for the computation are insufficient in that the calculation of the velocity components at a new network point using three known neighboring points, the determinacy requirement for hyperbolic equations is not satisfied. With the approximations made, two of the network points used lie on the fore cone of the point being calculated, but the third point is in a direction nearly perpendicular to the main flow, and so is well outside of the fore cone.

Joseph Sternberg, USA

535. L. G. Loitsyanskii, Generalization of the Joukovsky formula for the case of subsonic flow around a cascade of profiles (in Russian), Prikl. Mat. Mekh. 13, 209–216 (Mar.-Apr. 1949).

This paper presents a mathematical proof that the Joukovsky formula (according to which the lift force acting on a single blade is equal to the product of gas density, mean vectorial velocity of the gas ahead and behind the blade, and the circulation around the blade) can be extended to a cascade of blades in a compressible subsonic flow, if the gas density is taken as an arithmetical mean of the densities of the undisturbed gas ahead and behind the cascade.

P. Petroff, USA

536. Ira H. Abbott and Albert E. von Doenhoff, Theory of wing sections, New York, Toronto, London, McGraw-Hill Book Co. Inc., 1949, viii + 693 pp. Cloth, 9.2 × 6.2 in., 191 figs., $15.

This book presents that part of the theory of fluid mechanics which pertains directly to the behavior and use of wing sections at subsonic speeds, together with complete aerodynamic data for the currently important NACA airfoil sections which are designed for use in this flight range. Although intended primarily as a reference for the use of aerodynamicists and engineers, the rational and concise presentation of the essential elements of fluid flow and wing theory would appear to qualify this book as an excellent text or supplement for courses in airplane design and applied aerodynamics.

Starting with elementary considerations, the book first presents the basic and general notions of airfoil section and wing characteristics. Lifting-line theory is presented along with an introduction to the application of section data to wing design. This is followed by a brief discussion of the fundamental theory of two-dimensional fluid flow which includes a study of flow about a cylinder with circulation.

In preparation for the later chapters on airfoil theory and design, the fundamental theory is extended to basic airfoil-type sections with helpful digressions en route to present briefly the necessary mathematics of complex variables and conformal transformations. Modifications of the basic theory required by camber of the airfoil section, and the relation of camber to pressure distribution are thoroughly discussed.

A chapter is devoted to a rational treatment of viscosity effects and boundary-layer phenomena. This is followed by chapters on the classification of families of wing sections, experimental characteristics, high-lift devices (including boundary-layer control), and compressibility effects and corrections.

The second half of the book presents aerodynamic data on the NACA airfoils of the four- and five-digit groups, and of the 6- and 7-series forms.

John E. Goldberg, USA

537. E. A. Krasilshchikova, Disturbed motion of air caused by vibration of a wing moving at supersonic speed, Hdqtrs. Air Mat. Comm. Dayton Tech. Rep. no. 102 (Dec. 1949). See Rev. 1, **1034**.

538. M. I. Gurevich, Remarks on the triangular wing in supersonic flow, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1214-1A, 9 pp. (1949).

The problem of a thin flat triangular wing traveling at a supersonic velocity is investigated using the linearized conical flow method of Busemann. Several cases are considered using different

combinations of subsonic and supersonic edges, the choice of the edges being restricted by the condition that the resulting flow be conical. Hideo Yoshihara, USA

539. C. W. Frick and R. S. Chubb, The longitudinal stability of elastic swept wings at supersonic speed, Nat. adv. Comm. Aero. tech. Note no. 1811, 38 pp. (Feb. 1949).

It is assumed that the wing panels are elongated, so that beam theory is applicable for estimation of deflections. It is further assumed that the bending-deflection curve is parabolic and the torsional-deflection curve linear. This permits the loading due to elastic effects to be calculated, at least approximately, from published results of the linearized theory. The loading of the rigid wing, which is here assumed to be flat, is also available in published results. For the structurally-simplified wing treated here, the elastic-twist increments due to bending and torsion are proportional, and their ratio is given by the ratio of root bending and torsional moments. It is suggested that this ratio be estimated without including the loading due to torsional deflection; successive approximations can be made if necessary. The theory is worked out for wings with subsonic leading edges and supersonic training edges, but it is supposed to be extensible to supersonic-leading-edge wings as well. However, it is assumed, in calculating torsional moments, that the distance from center-of-pressure to elastic axis, in per cent of local streamwise chord, is a constant.

The results are presented in the form of ratios of rigid- to elastic-wing lift and moment slopes. The present theory has been applied to a wing with 60-deg leading edge sweepback at Mach number 1.414, for two dynamic pressures. The same ratios have been calculated by incompressible-flow theory for the same dynamic pressures. It is found that the ratios depend mainly on the dynamic pressure for this wing. The use of a modified strip theory is also discussed. W. R. Sears, USA

540. John W. Miles, A note on a solution to Possio's integral equation for an oscillating airfoil in subsonic flow, Quart. appl. Math. 7, 213–216 (1949).

The equation is

$$w(x, t) = (1/2\pi\mu) \int_{-1}^{1} \gamma(\xi, t) \exp[ik(\xi - x)](x - \xi)^{-1} R\, d\xi,$$

where R denotes $R[M, \mu^2 k(x - \xi)]$, defined by

$$R(M, y) = \tfrac{1}{2} i\pi M y \int_{-\infty}^{y} \exp(iu) H_1^{(2)}(M|u|)|u|^{-1} du,$$

where $H_1^{(2)}(z)$ is the Hankel function. Here $w(x, t)$ is the downwash along the chord of a thin oscillating wing (in this case proportional to $\exp(i\omega t)$), $\gamma(x, t)$ is the pressure, M is the Mach number, μ denotes $(1 - M^2)^{1/2}$, and k is proportional to ω. [See Küssner, Luftfahrtforschung 17, 370–378 (1940)]. The method used here is to expand the Hankel function in powers of M^2 (including terms in $M^{2n} \log M$), and finally to use the technique of solution employed by Schwarz in the incompressible case [ibid. 379–386 (1940)]. The author points out how this method can be improved by iteration, and carries out the calculation for the terms of order M^2, $M^2 \log M$. W. R. Sears, USA

541. G. Temple and H. A. Jahn, Flutter at supersonic speeds: derivative coefficients for a thin aerofoil at zero incidence, Rep. Memo. aero. Res. Counc. Lond. no. 2140, 35 pp. (1945, publ. 1949).

This report contains a derivation of the aerodynamic forces that act on an oscillating airfoil subject to the restrictions of two-dimensional supersonic flow over an infinitesimally thin airfoil at zero mean incidence executing simple harmonic oscillations of constant amplitude in vertical translation, pitch, and flap rotation. The method used involves the solution of the hyperbolic partial differential equation for the amplitude of the velocity potential by Riemann's method. Theoretical expressions which involve integrals of Bessel functions are developed for the aerodynamic lift and pitching moment. Equivalence is shown between these expressions and earlier work carried out both by Possio and by Borbely using different methods. The report includes numerical tables and graphs giving the aerodynamic lift and pitching moment coefficients for Mach numbers between 1.2 and 2.0 for values of the reduced frequency between 0 and 1.5. Mathematical formulas are also presented to permit somewhat simplified procedures for computing the aerodynamic coefficients for nontabulated values of the Mach number and reduced frequency.
Benjamin Smilg, USA

542. F. I. Frankl', The flow of a supersonic jet from a vessel with plane walls, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1213-1A, 7 pp. (1949) = Doklady Akad. Nauk SSSR 58, p. 381 (1947).

A more appropriate title would be "Flow of a supersonic jet in two dimensions." Essentially the problem is that of determining the velocity distribution and flow coefficient for maximum possible flow from a vessel, when the walls near the aperture make any assigned angle with the center line of the jet. A general method of solution is given, and numerical values are calculated for a particular case. Complete understanding of the paper requires reference to a previous paper by the same author which has evidently not yet been translated. C. W. Smith, USA

543. J. Nicholas and A. Benoit, Preliminary study of an air intake in two-dimensional supersonic flow (in French), Off. nat. Étud. Rech. aéro. Rep. no. 27, 31 pp., 2 charts (1949).

The paper describes supersonic-flow tests of an arrangement which resembles two-dimensional air intakes, and must be considered a preliminary study of the problems to be expected with actual air intakes. The walls of the intakes are formed by two profiles whose relative positions and angles of attack can be changed. The static and the Pitot pressure were measured by means of suitable pressure probes, and shadowgraphs of the flow patterns are given. Depending upon the configuration of the duct and the way in which the configuration is changed during the experiment one obtains different types of flow patterns. The results are compared with theoretically determined flow fields. Essential deviations were found and attributed to boundary-layer effects. The theoretical considerations include also flow patterns in which a normal shock arises along the axis of the duct. The method of computation, although it cannot be considered perfect, represents an interesting theoretical approach to a difficult phenomenon.
Gottfried Guderley, USA

544. Louis Viaud, Remarks on the optimum exit pressure in a ramjet (in French), Rech. aéro. Paris no. 8, 53–62 (Mar.-Apr. 1949).

The supersonic flow at the exit from ramjets is analyzed in terms of the static pressure in the discharge section. High thrust values are obtainable for pressures considerably below ambient (over-expanding nozzles) with corresponding high values of the form drag. For increasing discharge pressures a rapid decrease in thrust can be noted until the minimum thrust is reached at sonic discharge from the jet nozzle. A decrease in form drag accompanies the drop in thrust values. The optimum discharge pressure, i.e. highest net thrust, occurs at pressures below ambient, being in some cases 50% below ambient. The corresponding specific fuel consumption is always smaller than that of a ramjet operating with ambient pressure in the discharge section.
Andrew A. Fejer, USA

545. Harold Grad, Note on straight pipe jet motors, Commun. appl. Math. 2, 71–77 (Mar. 1949).

The author determines the thrust of a straight-pipe jet motor on the hypothesis that the burning producing chemical reaction takes place not at one point only but at several points of the tube axis. The reaction is assumed to take place instantaneously and the flow is assumed to be one-dimensional, i.e., uniform across the cross section of the tube. The author considers two cases as follows: (a) the sound speed is attained at the external section, in which case the author determines a boundary condition at the open end which the burning rate must fulfill; (b) the sound speed is attained at none of the sections, in which case the author imposes a boundary condition that the exit pressure be equal to the atmospheric pressure. In both cases the thrust is the same no matter whether the fuel is injected at several points of the axis or at one point only, provided the reaction is the same.

Carlo Ferrari, Italy

546. A. M. Faĭnzilber, Some cases of reduction of equations of motion in the boundary layer of viscous compressible fluid to ordinary differential equations (in Russian), Doklady Akad. Nauk SSSR 64, 775–778 (Feb. 1949).

In the present paper, the equations $\rho u u_x + \rho v u_y = -p_x + (\mu u_y)_y$, $\rho u = \Psi_y$, $\rho v = -\Psi_x$, $p = R\rho T$, $(\mu/\mu_0) = (T/T_0)^n$ of the boundary layer are reduced to ordinary differential equation. Here u and v are velocity components, ψ the stream function, ρ density, μ viscosity, T absolute temperature, p pressure, R the gas constant, and n a constant which depends upon the nature of gas. By introducing the variable $\eta = -nc_0^{-1}T_0^{-n}\,{}_0\!\int^u u\,du/T$ and denoting $\psi^{(1)} = [RT_0^{1/2}\mu_0^{-1}(2c_p)^{-1/2}]^{1/2}\psi$ the author obtains the equation

$$\psi_{\eta\eta}^{(1)} - p^{-1}\eta^{k-1}(1-\eta^{\kappa})^{1/2}\psi_x^{(1)}\psi_\eta^{(1)2} - [(\kappa-1)/k\kappa]p^{-2}(dp/dx)\,\eta^k(1-\eta^{\kappa})^{-1/2}\psi_\eta^{(1)3} = 0.$$

The author shows that the above equation has the same form as that for a viscous fluid and that only the coefficients differ. Assuming $\psi^{(1)} = [M(x)]^{1/2}B(\eta)$, he obtains for $B(\eta)$ an ordinary differential equation. Several cases where this assumption is permissible are discussed.

Stefan Bergman, USA

547. Luigi Crocco, The laminar boundary layer in gases (in Italian), Consiglio Naz. Ricerche. Pubbl. Ist. Appl. Calcolo 187, 78 pp. (1947) = Monografie Scientifiche di Aeronautica 3 (1946).

The laminar boundary-layer equations for a gas are transformed into two equations for τ, the viscous stress, and ρ, the density; with x, the coordinate along the surface, and u, the x-component of the velocity, as independent variables. These equations are solved fairly simply when the main-stream pressure is constant and either the Prandtl number is unity or the product of density and viscosity is constant. A recurrence method is given when neither of the last two assumptions holds. The second assumption is found to give much more accurate results for air than the first. Using it, the author is enabled to give a full description with many graphs of the laminar flow of gas past a flat plate.

Courtesy of Mathematical Reviews M. J. Lighthill, England

548. Dean R. Chapman, Laminar mixing of a compressible fluid, Nat. adv. Comm. Aero. tech. Note no. 1800, 19 pp. (1949).

A theoretical investigation is made of the velocity profiles for laminar mixing of a compressible-fluid stream with a region of fluid at rest assuming that the Prandtl number is unity. A method which involves only quadratures is presented for calculating the velocity profile in the mixing layer for an arbitrary value of the free-stream Mach number.

For the free-stream Mach numbers 0, 1, 2, 3 and 5, velocity profiles are presented for both a linear and a 0.76-power variation of viscosity with absolute temperature. The calculations for a linear variation are much simpler than those for a 0.76-power variation. It is shown that by selecting the constant of proportionality in the linear approximation so that it gives the correct value for the viscosity in the high-temperature part of the mixing-layer, the resulting velocity profiles are in excellent agreement with those calculated for a 0.76-power variation.

The velocity profiles for laminar mixing are calculated starting with zero boundary-layer thickness of an air stream of arbitrary temperature and a dead-air region also of arbitrary temperature. In cases where a laminar boundary layer of appreciable thickness exists at the point where mixing begins, the results given are not directly applicable in the initial part of the mixing region. For such cases, it is necessary to make some supplementary approximation in order to apply the results.

Nicholas Di Pinto, USA

549. L. E. Kalikhman, Heat transmission in the boundary layer, Nat. adv. Comm. Aero. tech. Memo. no. 1229, 43 pp. (Apr. 1949); transl. from Prikl. Mat. Mekh. 10 (1946).

The calculation of the thicknesses of the temperature and velocity boundary layers, the surface-friction coefficients, and the heat-transfer coefficients, is treated in an approximate manner for compressible supersonic and subsonic flows over both two-dimensional bodies and bodies of revolution. The calculation of the profile drag is restricted to two-dimensional bodies at Mach numbers for which shock waves are not present.

The two basic equations on which the approximate methods depend are obtained by integration of the boundary-layer equation of motion and the boundary-layer energy equation with respect to the distance from the surface. The integrated equations are given in a form in which the gas density does not appear explicitly, and which is the result of transforming the distance from the surface, y, to a new variable, η. The variable η is a generalization to the case of heat transfer of the transformation introduced by Dorodnitsfn [C. R. Acad. Sci. URSS 34, no. 8 (1942)] for the case of no heat transfer.

For laminar flow over two-dimensional bodies and over bodies of revolution, the surface friction, the heat transfer, the separation point, and the thicknesses of the velocity and temperature boundary layers are obtained by using the integrated boundary-layer equations of motion and energy together with the assumption that the profiles of the stagnation-temperature defect and the profiles of the velocity are expressible as fourth-degree polynomials in the variables η/δ and η/Δ, where δ and Δ are the thicknesses of the velocity and temperature boundary layers respectively. The assumption is made that the Prandtl number is unity.

For turbulent flow, the mixing-length concept is used to derive the logarithmic law for the velocity distribution and the distribution of the stagnation-temperature defect across the boundary layer. The distance from the surface is again expressed as a function of η. The logarithmic formulas for the stagnation-temperature defect and the velocity distribution are used with the integrated equations of motion and energy to derive equations from which the boundary-layer thicknesses and the surface distribution of the friction and heat-transfer coefficients can be calculated.

Finally, equations for the calculation of the profile drag are derived by using the integrated equations of motion and energy for the wake, in a development parallel to that of Squire and Young [Aero. Res. Counc. Rep. Memo. no. 1838 (1938)].

Neal Tetervin, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 536, 546, 547, 548, 549, 570)

550. Rudolf Iglisch, Exact calculation of laminar boundary layer in longitudinal flow over a flat plate with homogeneous suction, Nat. adv. Comm. Aero. tech. Memo. no. 1205, 69 pp. (Apr. 1949); transl. from Schr. Dtsch. Akad. Luftfahrtforsch.

For researchers interested in the problem of stabilizing the laminar boundary layer by suction, this report represents an impressive contribution. In the first portion of the report, the Prandtl boundary-layer equations for the case of arbitrary suction on a plate with no pressure gradient are transformed into a form suitable for numerical integration. The numerical procedures are carried through in considerable detail for a plate with uniform suction. The boundary-layer profiles are given for the successive stages as the profile changes from the Blasius profile at the leading edge of the plate to the asymptotic suction profile given earlier by Schlichting. The methods used are sufficient to give with acceptable accuracy the first and second derivatives of the velocity distributions. Plots are given of these derived functions. In addition the stream-line patterns are plotted as well as curves showing the various boundary-layer parameters such as skin function, momentum and displacement thickness and the like. This investigation was initiated in order to obtain velocity distributions that could be used in a study of the stability of laminar layers that have been preserved to great lengths by the application of suction. Such stability studies were undertaken in a separate report by A. Ulrich. F. H. Clauser, USA

551. W. S. Coleman, Analysis of the turbulent boundary layer for adverse pressure gradients involving separation, Quart. appl. Math. 5, 182–216 (1947).

The author proposes a method of calculating the development of a turbulent boundary layer and compares it with the partly empirical methods of von Doenhoff-Tetervin and of Garner. The present method is still based on certain assumptions regarding the conditions inside the boundary layer. The parameters occurring in the present scheme agree with those in Garner's equation, but the discrepancies between the three methods are not important in the final results. The author emphasizes the value of the present discussion in establishing the essential parameters, and states that "for ease and rapidity of calculation in practical cases, the empirical approach may prove more attractive." C. C. Lin, USA

552. Léon Agostini, The spectral function of the isotropic turbulence (in French), C. R. Acad. Sci. Paris 228, 736–738 (Feb. 28, 1949).

The author considers the spectral function $_{k_x}\int^{\infty} k^{-1}F(k)dk$, where k is the wave number and $F(k)$ is Heisenberg's spectrum. Usual results are discussed in terms of this function.

C. C. Lin, USA

553. L. B. Zarudnĭi and T. T. Usenko, Unsteady turbulence and the theory of flameless combustion (in Russian), Doklady Akad. Nauk SSSR 65, no. 5, 705–708 (Apr. 1949).

This article describes turbulent combustion of a hydrodynamically unstable flow of a combustible mixture moving in a channel. In this case, which is characteristic of flameless combustion, fundamental relations and factors are outlined for the beginning of steady process. It is shown that the criterion for the determination of the spreading of the turbulent flame consists not in the main velocity of the flow but the value of the mean quadratic pulsation velocity. L. M. Tichvinsky, USA

554. G. I. Taylor and G. K. Batchelor, The effect of wire gauze on small disturbances in a uniform stream, Quart. J. Mech. appl. Math. 2, 1–29 (Mar. 1949).

If a wire gauze is placed at an angle θ to an airstream, the pressure drop and the deflection of the stream ϕ are the important steady aerodynamic properties which determine the reduction of the turbulence. This paper investigates theoretically the effect of the gauze on spatial nonuniformity and on turbulence in the main stream. The main assumptions are: (1) the deviations from the main stream speed are small so that second-order terms can be neglected; (2) the Reynolds number of the flow past the gauze elements is low enough, so that no turbulence is generated by the gauze; (3) the gauze has aerodynamic properties which are rotationally symmetric about the normal to its plane; (4) in accordance with experiments by Simmons and Cowdrey and by Dryden and Schubauer, $\phi = \alpha\theta$ where α is a constant for a particular gauze and for small values of θ and ϕ.

The authors first investigate the effect of a gauze on a small steady disturbance superimposed on a uniform flow. The theoretical reduction in the disturbance is consistent with the few published measurements. There is one gauze with a resistance coefficient of 2.76 which completely damps all steady longitudinal disturbances; any lateral disturbances however remain appreciable.

For the analysis of turbulence reduction through gauzes the problem is again linearized, and therefore it is possible to find the effect of the gauze by studying disturbances of a single wave number, instead of dealing with the entire frequency spectrum. It is found that the longitudinal and lateral components are reduced by different amounts so that initially isotropic turbulence becomes axially symmetric after passage through the gauze. The results of the theory agree well with the experiments of Dryden and Schubauer for reduction of the lateral component; the agreement in the case of the longitudinal component is less good but better agreement could be expected if the measurements had been made nearer to the screens.

In an appendix Dryden and Schubauer analyze experimental results by Spangenberg at the National Bureau of Standards, in which the lateral force on wire screens is obtained. It is found that the force coefficient for the lateral force divided by θ is a function only of the coefficient of resistance of the screen.

A. M. Kuethe, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 513, 529, 536, 537, 538, 539, 561)

555. William T. Hamilton and Warren H. Nelson, Summary report on the high-speed characteristics of six model wings having NACA 65_1-series sections, Nat. adv. Comm. Aero. Rep. no. 877, 17 pp. (1949).

Results of a group of high-speed wind-tunnel tests of several model wings having NACA 65_1-series are presented. Thickness ratios of the several wings were from 8 to 12%, aspect ratios ranged from 7.2 to 10.8, and the tests were run at speed up to a Mach number of 0.9. The effects of thickness and aspect ratio variation upon aerodynamic characteristics and upon power requirements and economy at high subsonic speeds are discussed.

John E. Goldberg, USA

556. Edward C. Polhamus, A simple method of estimating the subsonic lift and damping in roll of sweptback wings, Nat. adv. Comm. Aero. tech. Note no. 1862, 20 pp. (Apr. 1949).

By judicially combining various corrections for lifting-line theory with empirical results, the author arrives at simple for-

mulas with aspect ratio and sweepback as parameters. The resulting increase of accuracy of estimates should be welcome to the designer. M. V. Morkovin, USA

557. M. M. Callan, A theoretical survey of induced vertical velocity in front of an aircraft wing, Nat. Res. Counc. Canada Aero. Note no. AN-5, 15 pp. (1949).

Numerical calculations for downwash from Glauert's formulas are presented in the form of charts useful in determining a suitable position for mounting an angle of attack indicator on a glider. The results permit the construction of charts showing the variation of downwash with angle of attack at an instrument fixed to the wing. Stephen H. Crandall, England

558. Leonard Sternfield and Ordway B. Gates, Jr., A method of calculating a stability boundary that defines a region of satisfactory period-damping relationship of the oscillatory mode of motion, Nat. Adv. Comm. Aero. Tech. Note no. 1859, 25 pp. (Apr. 1949).

This paper presents a method which extends the usefulness of the stability diagram as a tool in research and design. The stability diagram, as used in analysis of airplane lateral dynamics, suffers from a major defect. Routh's discriminant, equated to zero, plotted as a function of two design variables, defines combinations of these variables which insure oscillatory stability. The damping of a stable combination, however, will not be known. In particular, it is impossible to know immediately whether the system (airplane) will meet a given criterion of time to damp to half amplitude in terms of the period, as, for example stated in handling qualities specifications.

The note gives a method for constructing a boundary on the stability diagram which will define a region of satisfactory behavior for any required damping-period relationship. A chart is included for estimating the parameters involved in the construction of the boundary corresponding to the relationship at present required by the United States military services handling qualities specifications for the Dutch roll. This method requires less computation than the construction of lines of constant period and damping to give equivalent information concerning the adequacy of the damping for any stable combination of the design variables.

There is further presented a method for constructing lines of constant rate of divergence of the aperiodic motion. A method for determining on which side of any boundary the satisfactory region of the damping period relationship exists, and a method for extracting the roots of a stability quartic where Lin's method does not converge rapidly, are given in appendixes.

While intended to be a designer's tool, the stability diagram continues to be an important means of presenting the results of research on airplane dynamics. The method described of increasing the amount of graphical information it conveys should greatly enhance its value. Dunstan Graham, USA

559. Anshal I. Neihouse, Spin-tunnel investigation to determine the effectiveness of a rocket for spin recovery, Nat. adv. Comm. Aero. tech. Note no. 1866, 19 pp. (Apr. 1949).

New means for stopping a spin, in spin testing of new airplanes, are continually being sought. The usual method has been to open parachutes, either at a wing tip or at the tail. However, there are certain dangers involved in opening and releasing a spin-recovery parachute.

In this report, it is proposed to install a rocket at the tip of the wing(s) in order to provide sufficient yawing moment in an uncontrollable spin so that spin rotation will stop. The airplane will then nose down and pick up speed to a point where control can be regained. Tests of a freely spinning model equipped with such a device were made in the Langley 20-ft spin tunnel, and successful spin recovery was demonstrated. A rocket fired rearward from the inboard wing tip (right wing tip in a right spin) terminated the spin rapidly.

A discussion of the method of determining the expected total impulse of rocket required for a given airplane size and characteristics is included. E. Arthur Bonney, USA

560. F. W. S. Locke, Jr., Determining the hydrodynamic characteristics of flying boats, Aircr. Engng. 21, 104–112 (Apr. 1949).

The paper describes methods used to obtain quantitative data about: low-speed maneuverability, prehump directional stability, longitudinal stability in smooth water, main-spray characteristics, and rough-water landing behavior.

The purpose of the described tests is to determine the usable characteristics of a full-scale aircraft and the suitability of a given airplane for service use. It is not to correlate the obtained data with model tests.

The only special instruments required in the flying boat are a visual trim-angle indicator, an elevator-angle indicator, and an accelerometer. After a full description of the tests methods, tables containing the results obtained by conventional flying boats are given. F. C. Haus, Belgium

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 537, 539, 541, 566)

561. Franklin W. Diederich, Calculation of the aerodynamic loading of flexible wings of arbitrary plan form and stiffness, Nat. adv. Comm. Aero. tech. Note no. 1876, 52 pp. (Apr. 1949).

The paper presents a matrix method for calculating aerodynamic loading, the divergence speed, and certain stability derivatives of lifting surfaces of arbitrary plan form and stiffness. The assumptions are: (a) strip theory is applicable to calculate aerodynamic induction; (b) small deflections and small angle of attack; (c) the wing is mounted perpendicular to elastic axis; (d) elastic axis is straight in outboard wing and extended toward inboard; (e) all deformations are referred to elastic axis.

Provision is made in the method for using either stiffness curves and root-rotation constants, or influence coefficients. Computing forms, tables of numerical constants required in the analysis, and an illustrative example are included.

The method seems flexible enough to take account of more accurate knowledge on stability derivatives, and structural rigidity. It is useful to the practical designer.

Chieh-Chien Chang, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 419, 432, 433, 522)

562. W. Kruger, On wind tunnel tests and computations concerning the problem of shrouded propellers, Nat. adv. Comm. Aero. tech. Memo. no. 1202, 79 pp. (Feb. 1949).

The report is valuable because it correlates theory with systematic experiments, and arrives at fairly definite conclusions. The general objectives of the investigation are well set forth in the introduction. Propeller diameters, to obtain a sufficient static thrust, usually have to be larger than is necessary for high speed. Efficiencies in flight then do not reach their possible optimum value. With shrouded propellers considerable improvement of static thrust can be obtained. A shroud is, therefore, a

means in extending the power limits of the normal propeller.

The report gives test results on a shrouded propeller designed for a high ratio of advance and relatively high thrust loading. The forces at the propeller and at the shroud were measured separately. The detailed investigations also permitted the inclusion of the influence of energy losses upon the aerodynamic behavior of the shrouded propeller.

The model consisted of an ellipsoid-shaped nacelle. In the nacelle there was a high-speed a-c, 30-hp, 30,000-rpm motor. A variety of shroud models was tested in combination with propellers having different number of blades.

The theoretical considerations cover: *Propeller in forward motion:* (1) Ideal case without energy losses; (2) shrouded propeller with energy losses taken into consideration; (3) calculation of the additional energy loss caused by the additional velocities for propeller in motion; (4) calculation of the mass flow flowing through the propeller; (5) calculation of the pressure jump to be established; (6) the operating condition of the propeller for varying ratio of advance. *Propeller at rest:* (1) Ideal case without energy losses; (2) shrouded propeller at rest with energy losses taken into consideration.

Besides the usual force measurements and visualization tests, additional measurements were also made concerning the problem of the slip-stream cross section and the pressure losses in the flow through the shroud.

The conclusions, briefly summarized, are: When a propeller is heavily loaded, a stator is absolutely necessary in order to avoid the high rotational losses and the large pressure drag of the nacelle due to suction at the afterbody. The maximum efficiency measured was 0.71. The measurements permit the conclusion that maximum efficiency could be essentially increased by using shroud forms of smaller chord and profile thickness. The shrouded propeller surpasses the standard propeller by essentially better static-thrust factors of merit. Alexander Klemin, USA

563. Harvey H. Hubbard and Arthur A. Regier, Free-space oscillating pressures near the tips of rotating propellers, Nat. adv. Comm. Aero. tech. Note no. 1870, 64 pp. (Apr. 1949).

The theory is given for pressures associated with a rotating propeller, at any point in space. Because of its complexity, this analysis is convenient only for use in the critical region near the propeller tips where the assumptions used by Gutin [Phys. Z. Sow. Un. 9, 1936] to simplify his final equations are not valid. Good agreement was found between analytical and experimental results in the tip Mach-number range 0.45 to 1.00.

Pressure charts based on experimental data are included for the fundamental frequencies of two-, three-, four-, five-, six-, and eight-blade propellers and for a range of tip clearances from 0.04 to 0.30 times the propeller diameter.

As the tip clearance is decreased, pressures in a region about as wide as one-propeller radius are greatly increased. The fundamental frequency of pressure produced by a four-blade propeller is essentially independent of tip Mach number in the useful tip Mach-number range. At tip Mach numbers near 1.00 so much energy appears in the higher harmonics that the total pressures produced by a two-blade propeller are only slightly greater than those produced by a four-blade propeller at the same tip Mach number and power coefficient.

Blade plan form is shown not to be a significant parameter; however, the nondimensional parameter, tip clearance divided by propeller diameter, is shown to be significant.

From authors' summary by G. Kuerti, USA

564. R. W. L. Gawn, Cavitation of screw propellers, Trans. N.E. Coast Instn. Engrs. Shipb. 65, 339–374 (Apr. 1949).

The paper discusses the analyses of cavitation being made in England from ship trials and work with the propeller tunnel at Haslar. Typical examples are discussed. Of particular interest is the development of new parameters for the plotting of cavitation data in which it appears that the cavitation characteristics of a large variety of propellers can be represented by a few simple curves. Although a broader application would be required to prove the reliability of this method of plotting, it would appear to have valuable possibilities.

The paper includes, as an appendix, a section describing the methods used in model-propeller production. F. E. Reed, USA

565. J. Balhan, An investigation of the applications of profiles with constant pressure distribution to ship propellers (in Dutch with English summaries), Ingenieur 's-Gravenhage 61, W19-25 (Apr. 29, 1949).

The author proposes to avoid propeller cavitation by using blade sections which give constant pressure on the suction side, thus avoiding the peak in the pressure curve. The advantages are summarized and a design with given data is discussed. (The author refers to the long calculations involved. He seems to be unaware of the simple direct method explained in the reviewer's *Theoretical aerodynamics*, pp. 144–153.) This design is compared with the hydrofoil type of usual shape.

L. M. Milne-Thomson, England

Flow and Flight Test Techniques

(See also Revs. 406, 554, 564)

566. C. C. L. Gregory, Theory of a loop revolving in air, with observations on the skin-friction, Quart. J. Mech. appl. Math. 2, 30–39 (Mar. 1949).

The problem of dynamic equilibrium and stability of a flexible loop, driven at constant speed about an axis normal to its plane by a small pulley, is solved. The shape and position of the loop are shown to depend only on the ratio of the aerodynamic-drag force to the gravity force, and the angle of the loop at the point of tangency with the pulley.

The author suggests the phenomenon as a means of measuring air friction on a moving belt or similar surface. Some simple experiments by him seemed to yield plausible results, albeit no mention is made of scale effect (i.e., Reynolds number). For relatively flat belts, the reviewer would expect that aerodynamic lifting forces might come into play. John W. Miles, USA

567. S. J. G. Taylor, The "ring balance" flowmeter, Metallurgia 39, 305–308 (Apr. 1949).

The ring balance has been used extensively as a flowmeter and the theory of this instrument is well known. The practical implications of this theory are reviewed and conclusions are drawn regarding the construction and operation of the meter. The various possible errors are analyzed and discussed.

André L. Jorissen, USA

568. G. M. Lilley and D. W. Holder, Experiments on an induction type high speed wind tunnel driven by low pressure steam, Coll. Aero. Cranfield Rep. no. 24, 16 pp., 11 figs (Mar. 1949).

The flow of air through the working section of 2.25-in. diameter up to Mach number of 1.7 was induced by a flow of steam (up to 120 psi) through an annular injector. The tunnel has low noise level and could be utilized by engineering colleges as a simple and inexpensive means of achieving high-speed flow for experimental

purposes. An approximate theoretical analysis of the performance is developed in the article. A. Petroff, USA

Thermodynamics

(See also Revs. 544, 576)

569. **Claude Fouré, The problem of combustion of a liquid fuel in a turbojet chamber** (in French), Rech. aéro. Paris no. 8, 5–8 (Mar.-Apr. 1949).

The two conventional systems are described: direct injection and prevaporization. The study of the fuel history from the tank to the combustion chamber should, according to the author, lead to means of increasing the efficiency of the engine.

The author refers to his very complete report (in collaboration with L. Reingold) on British research of this problem [Études et recherches britanniques sur les chambres de combustion de turboréacteur et sur la combustion, Off. nat. Ét. Rech. aéro. no. 2 (1948)]. D. Jacovleff, Belgium

570. **B. L. van der Waerden and J. Korevaar, Evaporation into a turbulent atmosphere** (in Dutch), Math. Centrum Amsterdam Rapport ZW 1948-006, 6 pp. (1948).

Let air be moving parallel to the surface of a strip of water and in a direction perpendicular to the edges of the strip. The authors propose to determine the rate at which the water evaporates due to the action of turbulence in the air stream. The motion of the air is steady, and the velocity gradient is proportional to log $(z + z_0)/z_0$, where z is the distance above the water and z_0 represents a friction coefficient at the interface. An exact solution by the Laplace transformation, similar to the solution of the heat equation, was attempted but could not be carried through. Instead, an approximate solution, also using the Laplace transformation, is presented and the validity of the approximation discussed.

Courtesy of Mathematical Reviews W. J. Nemerever, USA

571. **George Jaffé, A statistical theory of liquids, III,** Phys. Rev. (2) 75, 184–196 (1949).

The general kinetic equation is solved by using a series of simplifying assumptions. Numerical values are obtained for the heat conductivity and viscosity of ten liquids with the help of the intermolecular potentials previously used by the author [same source, (2) 62, 463–476 (1942); 63, 313–321 (1943)] for obtaining the equilibrium constants of these liquids.

Courtesy of Mathematical Reviews L. Tisza, USA

572. **E. B. Greenhill and J. R. Whitehead, An apparatus for measuring small temperature changes in liquids,** J. sci. Instrum. 26, 92–95 (Mar. 1949).

The apparatus described was developed by the authors to measure the very small heat quantities released when fine powders are immersed in various liquids. By altering the method of calibration, very small temperature changes could be determined. In combination with strain gages a similar technique could be employed to measure small changes in stress.

A thermistor forming part of a Wheatstone bridge gives the primary indication. The galvanometer mirror reflects a different amount of light towards a photoelectric cell. By interposing a rapidly rotating shutter between mirror and cell, the cell is excited to give an alternating current which is amplified and fed back to a second thermistor in the bridge circuit giving, in effect, a "null" method of measurement.

At the same time a recording instrument graphs the amplified current, thus providing a temperature record. In this instance the apparatus was directly calibrated to measure heat units. Sensitivity was of the order of $^1/_{60}$-calorie corresponding to a temperature change of 0.0002 C with the calorimeter used.

R. E. Button, South Australia

Heat Transfer; Diffusion

(See also Revs. 418, 505, 549, 571)

573. **K. R. Wilkinson and J. Wilks, Some measurements of heat flow along technical materials in the region 4 to 20 K,** J. sci. Instrum. 26, 19–20 (Jan. 1949).

Average values for the thermal conductivities of nickel silver, cupronickel, stainless steel, copper, and glass were measured in the range 4 to 20 K. One end of a specimen was held in boiling helium while the temperature of the other end was varied. The rate of helium evaporation provided a measure of the heat flow. Values of thermal conductivity at various temperatures were deduced. Myron Tribus, USA

574. **N. S. Billington, The thermal diffusivity of some poor conductors,** J. sci. Instrum. 26, 20–24 (Jan. 1949).

Angstrom's method for measuring the thermal conductivity of metallic bars is applied to the determination of the thermal diffusivity of poor conductors of heat. The results are compared with values which would be obtained from the separate measurements of conductivity, density and heat capacity, as reported in the literature. Myron Tribus, USA

575. **Helmuth Kühne, Laminar or turbulent flow for heat exchangers** (in German), Z. Ver. dtsch. Ing. 91, 154–156 (Apr. 1949).

There are two different values of internal tube diameters which have special significance for heat-exchanger design. One of these diameters, d_{cr}, corresponds to the lower limit of the turbulent regime. The other significant diameter is the value, considerably smaller than d_{cr} for which laminar flow gives the same heat transfer for a given pressure drop, as turbulent flow does with d_{cr}. The range between these two values gives relatively poor heat transfer. This paper derives expressions for these two diameters in terms of parameters thought to be convenient for heat-exchanger design. R. Hosmer Norris, USA

576. **Klaus Clusius, The thermal effect during diffusion as a lecture demonstration** (in German), Helv. phys. Acta 22, no. 2, 135–141 (1949).

When one gas diffuses into another, the temperature of one gas may decrease, while that of the other increases. The inverse effect also occurs, namely, a temperature field in a mixture of gases gives rise to a diffusion in such direction as to offset the temperature field. For H_2 and N_2 the maximum temperature change is 7.5 C at room temperature. An apparatus for demonstrating this effect is described. Proof is given that this effect does not depend on the Joule-Thomson coefficient or deviations from perfect gas laws. Myron Tribus, USA

Acoustics

(See also Rev. 563)

577. **Benson Carlin, Ultrasonics,** McGraw-Hill Book Co. Inc., New York, 1949, 270 pp. Cloth, 9.25 × 6.5 in., 160 figs., 3 tables, $5.

This book presents an outline of nondestructive testing by means of ultrasonics, and some material relevant to possible

industrial applications (coagulation, agitation, etc). The use of ultrasonics as a tool for research on the properties of matter, perhaps the most important application to date, is hardly discussed at all.

The various laboratory arts of interest to the physicists and engineers working in the field are discussed in a general way. A similar lack of detail limits the usefulness of the sections on electronic instrumentation. Nevertheless engineers and technicians concerned with ultrasonic testing will find the book a convenient summary of the art. There is little in the book of interest to the physicist. The typography is good and the illustrations are clean and well drawn. Martin Greenspan, USA

578. Uno Ingård, Characteristics of sound waves in a circular tube (in Swedish), Tekn. Tidskr. 79, 269–274 (Apr. 9, 1949).

The author consider the radiation into a circular tube from a plane piston. The solutions of the wave equation, and the radiation impedance are given for an infinitely long and a finite tube. The results derived are applied to a cylindrical resonator, yielding a modification of the usual formula for the resonance frequency. Some curves of measured and calculated resonance frequencies show good agreement. Fritz Ingerslev, Denmark

579. William J. Jacobi, Propagation of sound waves along liquid cylinders, J. acoust. Soc. Amer. 21, 120–127 (Mar. 1949).

This paper considers the natural modes of propagation along liquid cylinders, with various nondissipative boundary conditions and for the cases where the wave length of the natural frequency is comparable with the diameter of the cylinder. With the assumption of a nonviscous fluid with cylindrical symmetry, a velocity potential exists and the wave propagation along the radius is given by certain Bessel functions dependent on the particular boundary.

The field patterns, phase velocities, and cutoff frequencies are calculated for the simple natural modes of propagation in the following cases of liquid cylinders: (1) with rigid wall, (2) with pressure-release walls (zero pressure on the walls), (3) embedded in an infinite liquid medium of different density and sound velocity (it is found that the sound field tends to concentrate within the cylinder as the frequency increases), (4) inside another liquid cylinder with pressure-release wall, (5) with thin elastic walls.

These solutions can be applied to an arbitrary frequency distribution where the characteristic functions are orthogonal. Experimental results are presented for cases (2) and (5) above. It is shown that the measured values of phase velocity are in agreement with those predicted by the theory.

Chieh-Chien Chang, USA

580. R. D. Fay, Interactions between a plate and a sound field, J. acoust. Soc. Amer. 20, 620–625 (Sept. 1948).

When a plane acoustic wave in water strikes obliquely a submerged metal plate, a strong nonspecular "reflection" may occur back along the direction of incidence. The phenomenon is attributed to reradiation by a flexural vibration of the plate which is initiated by the incident wave and then reflected back from the plate boundary. To analyze the explanation, the equations of flexural vibrations in a "free" plate are augmented by terms representing forces due to the surrounding water. The equations are solved for the steady state, under assumption of constant propagation constants for the various modes, and yield three, instead of the usual two, flexural modes. These three are related to the two free modes by arguments of plausibility, and conditions are discussed for large vibrations of the type capable of reradiating and transmitting strongly. The angle of incidence must be near the critical angle. A criterion is given in terms of the product of acoustic frequency f and plate thickness b: $fb > 20$ kc inch for strong reradiation along the direction of incidence. (This paper is clarified by a companion article; see following review.)

A. O. Williams, Jr., USA

581. W. J. Finney, Reflection of sound from submerged plates, J. acoust. Soc. Amer. 20, 626–636 (Sept. 1948).

The nonspecular reflection sometimes found when a plane acoustic wave strikes a plate obliquely (see preceding review) is described qualitatively. Measurements are reported, made on steel plates about 1 in. thick and 14 in. square, submerged in water. Frequencies were 18–60 kc. Interrelations of frequency, incidence angle, plate thickness, amplitude of flexural vibrations of the plate, and magnitude of the nonspecular reflection were studied. Pulses about 1 millisec long were used to separate the various effects. The steady state was attained in less than this time. Mechanical details of the apparatus and acoustic units are described; the electronics is sketched. The general agreement with Fay's calculations was good. An empirical relation was found, relating incidence angle φ_0, plate thickness b (inches), and frequency f(kc) for maximum vibration of the plate and simultaneously for maximum nonspecular reflection: $\sin \varphi_0 = 2.16\, bf^{-1/3}$. There is some evidence that this relation applies to plates of other materials, and also in much different frequency ranges. A. O. Williams, Jr., USA

582. C. W. Kosten, Sound absorption by porous materials, II, Appl. sci. Res. Sec. B, vol. B-1, no. 4, 241–250 (1949).

In analogy with Kirchhoff's correction to the velocity of sound in a wide tube, the correction in the case of a narrow tube is considered, in which the flow is essentially of the Poiseuille type. Instead of the normal thermal conductivity an apparent conductivity, 0.5 to 0.2 of the normal, is introduced. The expressions derived are used to determine the sound-absorption coefficient of porous materials.

A consequence of the general theory is that the sound absorption coefficient of all materials with a higher porosity than 60 will be above 0.12 $(\nu l)^2$, ν being the frequency and l the thickness of the material. Fritz Ingerslev, Denmark

Ballistics, Detonics (Explosions)

(See also Rev. 532)

583. P. Curti and F. Dubois, The solution of the principal problem of exterior ballistics by means of a mechanical analogy (in German), Schweiz. Bauztg. 67, 52–54 (Jan. 15, 1949).

A brief description is given of a differential analyzer designed specifically for solving the equation of motion of the center of gravity of a projectile, considered as a point mass. The coordinates of the plane trajectory as well as the hodograph (projectile speed vs. inclination of the trajectory) are plotted mechanically on paper and the curves are marked at equal intervals of time. Numerical values of time, angular direction of motion, projectile speed, and the vertical and horizontal coordinates of the trajectory are recorded photographically. The air resistance is considered to be a function of the Mach number and of the air density, itself a function of altitude and temperature.

Four spherical integrators are used to solve simultaneously the four first-order differential equations of motion. Division and multiplication operations required to compute the Mach number and the resistance of the air are accomplished by deformable similiar triangles. Various shafts of the instrument are driven by electronically controlled motors.

Trajectories of rockets can be computed by manually adjusting several controls as the machine is operated. R. L. Pigford, USA

584. H. Kolsky, J. P. Lewis, M. T. Sampson, A. C. Shearman, and C. I. Snow, Splashes from underwater explosions, Proc. roy. Soc. Lond. Ser. A, 196, 379–402 (Apr. 7, 1949).

The experiments were carried out in a sheltered pond about 200 m long, 80 m wide and 11 m deep. The splashes from the underwater explosions of 1- and 10-lb spherical charges of P.E. no. 2 and Nobel's explosive "808" at various depths were photographed with high-speed motion-picture cameras. The phenomena at the surface of the pond were studied in greater detail than in previous investigations, and a number of new phenomena have been observed.

Measurements of the initial velocity of rise of water were made, and it is shown that for the range of depths $0.3 < W^{1/3}/D < 4$, where W is the weight of the charge in pounds and D is the depth in meters, the initial velocity V of rise of the water in meters per second is given to an accuracy of about 10% by

$$V = 66\,(W^{1/3}/D - 0.1).$$

This relation in conjunction with an empirical relation between V and the peak pressure P of the shock wave determines the constant C in the theoretical relation $P = C\,W^{1/3}/D$. The peak pressure of the shock can thus be given in terms of the distance from the charge.

For calm water and for values of $W^{1/3}/D$ between 1 and 0.3, the contour of the dome of water above the center of explosion is found to obey the relation

$$H = B\left(\frac{D^2}{D^2 + R^2} - A\right)$$

where H is the height of a point at a distance R from the center of the splash, A and B being constants depending on the nature of the charge.

The high-speed photographs registered a new phenomenon, i.e., that during the first few milliseconds after detonation there was a bright circular patch expanding rapidly over the surface of water. The radius was found to increase linearly with time with a speed nearly that of sound in water. This phenomenon is provisionally called a "crack."

The "black ring" around the dome was also photographed. This was found to consist of a roughening of the water, resulting from the intensification of ripples already present before the explosion, and from the formation of individual "spikes" in water which was originally very calm. The role played by the bubble of gas formed by the explosion is also considered. Y. H. Kuo, USA

Soil Mechanics, Seepage

(See also Revs. 504, 517)

585. Kano Hoshino, A fundamental theory of plastic deformation and breakage of soil, Proc. int. Con. Soil Mech. Found. Engng. 1, 93–103 (June 1948).

The author presents a theory of plastic deformation and soil failure based on the assumption that the two coefficients existing between stress and strain (one expressing the form change due to shearing stresses, and the other the volume change due to normal stresses) are both proportional to the strain energy stored in the soil. He derives expressions for these coefficients for the cases of normal stress with no shear and pure shear, in both two and three dimensions. The results are shown to compare well with experimental values. The stress condition at soil failure is obtained as the strain energy reduces to zero. The author infers that the relation between breaking stresses under simple compression tests simple shear tests and triaxial tests can be explained by these equations. E. Vey, USA

586. M. Duriez, Note on new methods to increase the stability and impermeability of the soil (in French), Ann. Ponts Chauss. 119, 289–295 (Mar.-Apr. 1949).

This article contains a brief description of the commonest methods of soil stabilization, namely: injection of bituminous emulsions, cement, and silicate of soda, treatment by electro-osmosis, electrochemical hardening and the freezing method. Ed.

587. M. A. Lukomskaya, On the flow of a fluid through a hole in an inhomogeneous layer (in Russian), Prikl. Mat. Mekh. 12, 207–208 (Mar.-Apr., 1948).

The author solves a problem of steady flow of a perfect incompressible fluid subject to specified boundary conditions. Given two (or more) regions D_1 and D_2 occupied by the fluid, it is required to find the characteristic function of flow

$$W_1(Z) = \varphi_1 + i\psi_1$$

in the region D_1 and the characteristic function of flow

$$W_2(Z) = \varphi_2 + i\psi_2$$

in the region D_2 subject to conditions $\psi_1 = \psi_2$, $\varphi_1/C_1 = \varphi_2/C_2$, C_1 and C_2 constant. The general case, treated first, assumes for the boundary separating D_1 and D_2 any analytic curve L. From this is deduced a particular case in which L is an ellipse. A. W. Boldyreff, USA

588. Leonard Schiff and F. R. Dreibelbis, Infiltration, soil moisture, and land-use relationships with reference to surface runoff, Trans. Amer. geophys. Un. 30, 75–88 (Feb. 1949).

This paper presents a method for determining rates of water movement in the soil under natural conditions. The method uses infiltration curves as the supply of water to the soil, and field measurements of soil-moisture changes. The relationships between infiltration, soil moisture and surface runoff were determined for two soil types in three vegetal covers.

The amounts of water taken up, at different times, by successive 1-in. layers of soil during storms were established by soil moisture measurements. Curves are given showing the rates of infiltration, rates of topsoil storage depletion, and rates of percolation into the top of the subsoil, all vs. time.

In case of the most severe storm, topsoil transmitted water at an average rate of 10 in. per hr through 7-in. topsoil for some 45 min until water reached the subsoil. Thereafter the rate diminished rapidly as topsoil storage became exhausted, and the rate of percolation into the subsoil became the governing factor. As topsoil became saturated, the percolation dropped to less than 0.1 in. per hr. Topsoil storage is therefore of tremendous importance. If runoff occurred, it generally began about the time water reached the subsoil.

Important points derived from the analysis are: (1) For soils of type studied, runoff rarely occurs before topsoil storage space is practically exhausted. (2) Topsoil can absorb a high rate of rainfall provided initial soil moisture is not too high. (3) Water reaches the subsoil shortly after runoff begins, after which topsoil saturation is rapidly approached and infiltration rates are materially reduced. (4) Subsoil storage in the soils studied was not effective in reducing runoff. (5) Frost seems to reduce initial infiltration. (6) Transpiration is important. Cover which exhausts soil moisture early in storm season provides greater storage

space, thus less runoff. Wheat is better than corn. (7) Soil exposure is important. Exposed soil is more rapidly sealed by washing and produces more runoff. (8) Subsoil is the bottleneck that limits infiltration. Deep topsoil with all-season all-over cover helps to limit runoff. Opening the subsoil as by deep-rooted plants or by mechanical means might also help.

John C. Geyer, USA

589. **L. Bendel, Influence of soil-water-level oscillations on the lifting and sinking of buildings and soil surfaces** (in German), Z. Öst. Ingen.-Archit.-Ver. 94: Mar. 4, pp. 39–42; Apr. 4, pp. 54–57 (1949).

The creation of numerous artificial lakes has raised many new problems of which two of the most important are (1) the relation between underground water conditions and the motion of the soil surface and of the buildings; (2) the connection between the oscillations of the lake level and the soil-water level or the critical soil-water slope, i.e., the slope which can produce mechanical erosion of the ground.

The author, a well-known specialist in engineering geology, gives data for such variations observed during a period of 3 years at the lake of Lugano, for water-level oscillations of 2.5 meters. In a condensed form, these relations are represented by means of graphs and tables, and a brief mathematical interpretation of the obtained results is given. A short description of the employed methods and apparatus completes this account. The conclusion is that a close relation between lake and soil-water level exists, with corresponding effects on the soil level.

Aurel A. Beleş, Rumania

Geophysics, Meteorology, Oceanography

(See also Revs. 588, 589)

590. **Sekizi Ogiwara, On the temperature of the air and the growth of cloud particles in convective clouds** (in Japanese), J. met. Soc. Japan 27, 15–20 (Jan. 1949).

The relation between the air temperature and the drop size in an adiabatically ascending cloud is studied theoretically. Some aspects of the structure of the convective cloud are elucidated.

S. Syōno, Japan

Lubrication; Bearings; Wear

591. **H. Stäger, Friction and lubrication** (in German), Schweiz. Arch. 15, 97–116 (Apr. 1949).

The paper deals with lubrication and application of liquid lubricants and coolants in metal forming and cutting, and concentrates on the chemical processes involved.

The effect of various additions (aromatic and aliphatic phosphates, chlorides and sulphides) to oil used for high-pressure lubrication is examined. The inadequacy of the conventional lubricant-testing machines is pointed out, and the results of service-bearing tests using the Timken-Allen machines (after L. L. Davis, Bert H. Lincoln and B. E. Lilley) are reported for various bearing metals and lubricant additions.

In liquid-jet chipping and cutting processes the additions produce lubricating and cooling layers by chemical action, replacing the here absent hydrodynamic action. Tests of the cutting force and the surface quality obtained are related (they include those by H. Ernst and E. Merchant).

For cold-working, a temperature-resistant and very hard phosphate layer acts as the carrier of the lubricant proper, and spectacularly increases the formability. By this method piano wire can be produced of tensile strength 280 kg/mm² after 15 draws with a 55-fold length increase.

The reviewer does not quite agree with some of the author's contentions, e.g., those concerning the effect of the orientation of the lubricant molecules, or the effect of "tensile viscosity" on friction. But these are admittedly debatable questions. Equations (1) to (4), concerning dry friction, contain contradictions, but fortunately they are not used in the sequel.

Hans Drescher, Germany

Marine Engineering Problems

(See also Rev. 564)

592. **Anders F. Lindblad, Some experiments with models of high-speed ships,** Shipbuilder 56, 318–324 (Apr. 1949).

Twenty-foot models of a 400-ft ship of 56-ft breadth and 23-ft draft were studied at speed-length ratios between 0.8 and 1.0. For block coefficients between 0.58 and 0.62 the minimum resistance was found for the center of buoyancy located 2 to 3% aft of $L/2$. The effects of individual variation in the fore and after body fullness on the resistance are reported.

J. M. Robertson, USA

593. **N. H. Burgess, Tanker design from a stability point of view,** Trans. N. E. Coast Instn. Engrs. Shipb. 65, 193–223 (Feb. 1949).

A study is made of the design of tankers as influenced by requirements of static and dynamic stability. The author compares the dimensions of modern tankers with those of tankers built 50 years ago and states what he considers to be the minimum requirements for stability of a tanker. He presents a procedure for making preliminary computations of stability in obtaining the over-all dimensions of a tanker.

The remainder of the paper deals with the effect of the subdivision of tank space and the combined effect of erections such as the poop, bridge, and forecastle. A comparison is made between tankers having two longitudinal bulkheads and tankers having single bulkheads. Curves are given showing the influence of erections and the effect of the free surface of liquid cargo on the statical stability of a typical 12,000-ton tanker. The author suggests that a reduction be made in the subdivision of tank space by the adoption of one longitudinal bulkhead instead of the two as now commonly used. He states that the requirements of static and dynamic stability would be satisfied for most tankers having single bulkheads. Some discussion is also given of the influence of this adoption on the requirements of strength of a tanker.

Frank Baron, USA